Local stability analysis of microwave circuits

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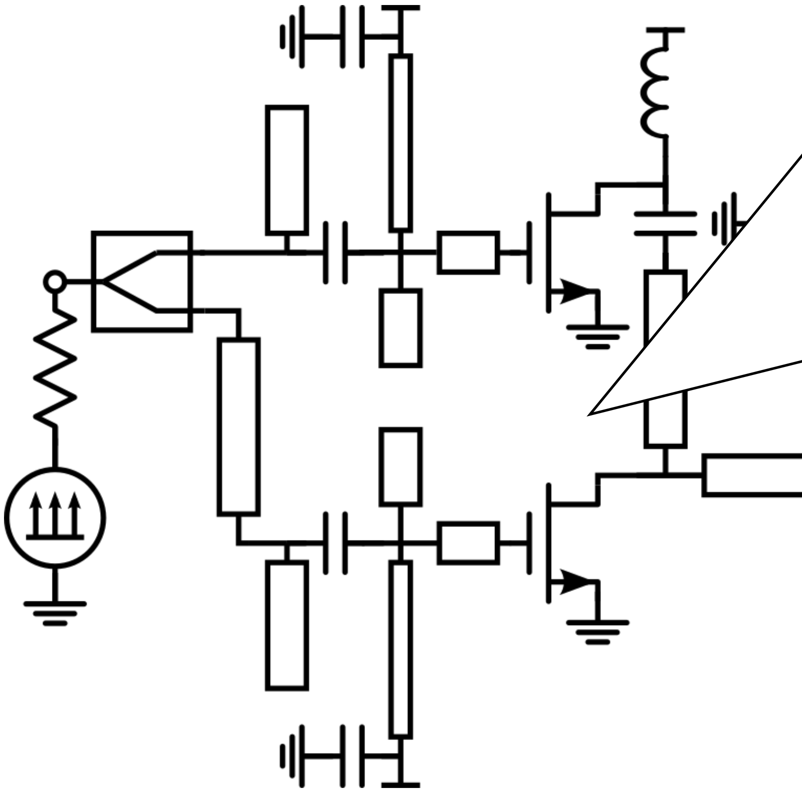
Modern circuit simulation tools



Frequency domain simulation methods

KEYSIGHT TECHNOLOGIES	ADS
NATIONAL INSTRUMENTS	MWO
cādence	Spectre

Proprietary simulator
Proprietary models

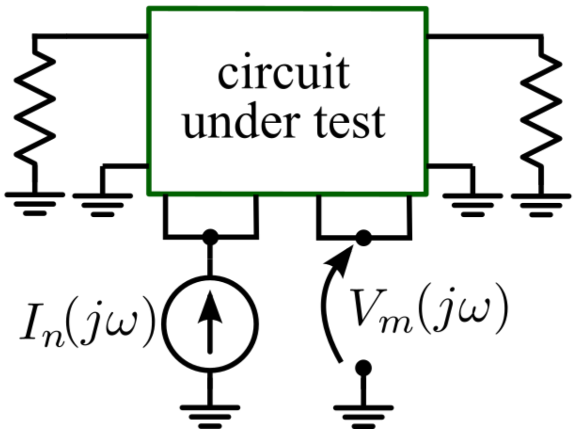
Equilibrium Solution
DC simulation

Periodic solution
Harmonic Balance

Local stability analysis: Linearise

Equilibrium Solution

DC simulation

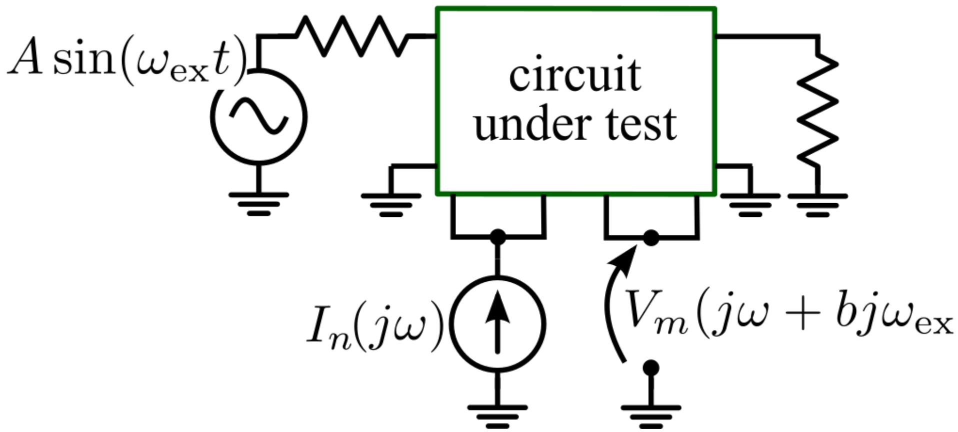


$$Z_{mn}(j\omega) = \frac{V_m(j\omega)}{I_n(j\omega)}$$

AC analysis

Periodic solution

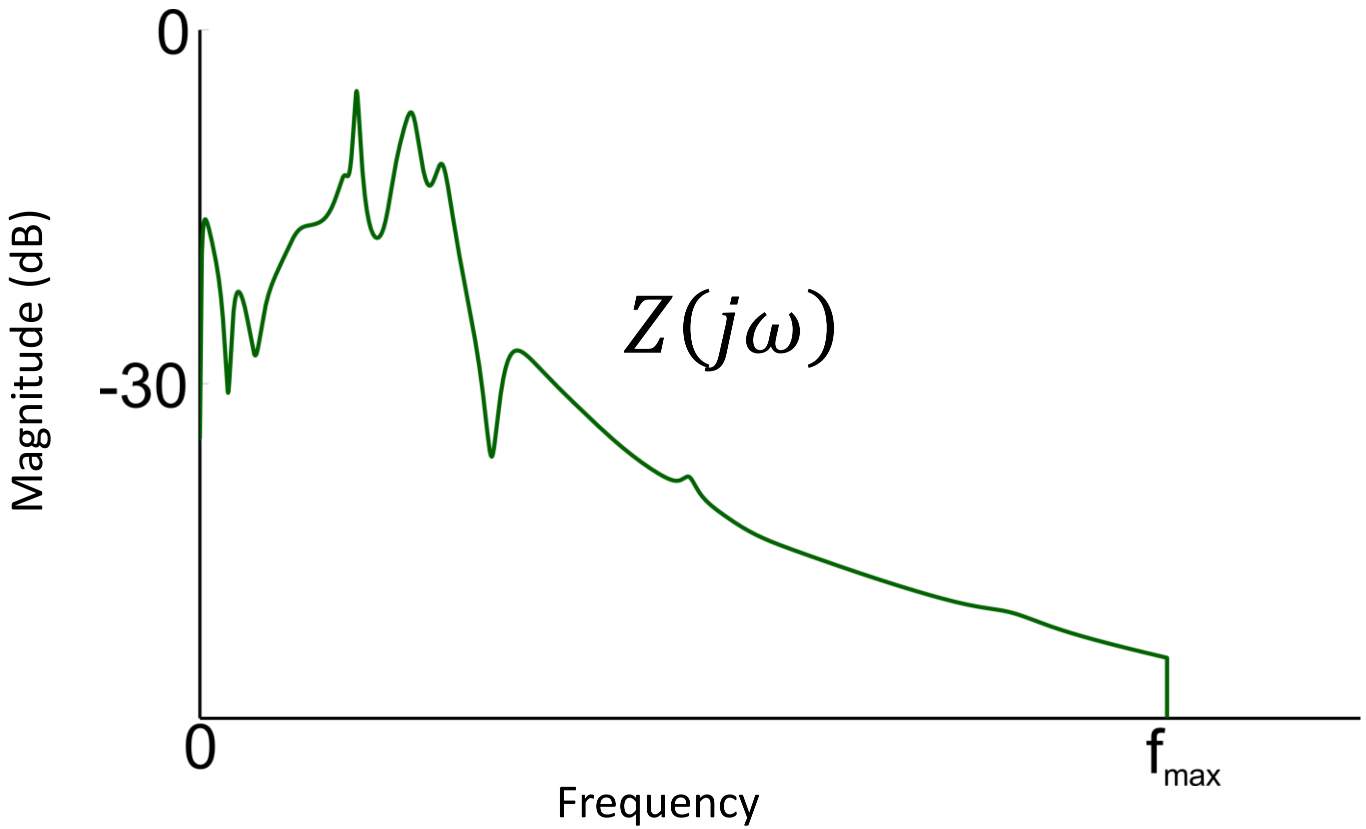
Harmonic Balance



$$Z_{mn}(j\omega) = \frac{V_m(j\omega + bj\omega_{ex})}{I_n(j\omega)}$$

LSSS analysis

Is frequency response stable?



$Z(j\omega)$ known on a discrete set of frequencies

Does $Z(j\omega)$ have right half-plane poles?

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Stability Analysis by projection

Examples

Filter influence

Estimating unstable poles

Assumptions:

Circuit contains delay $\Rightarrow Z(j\omega)$ meromorphic

Circuit is realistic $\Rightarrow$ unstable part rational

Unstable pole observable in $Z(j\omega)$

$Z(j\omega) \in \mathcal{L}_2$ $\Rightarrow$ No poles on the $j\omega$ Axis

Noiseless data

Stability in Hardy context

$$g \in \mathcal{H}_2 \; when \begin{cases} g \; analytic \; in \; RHP \\ \int |g(j\omega + \sigma)|^2 d\omega < \infty \quad \sigma \to 0 \end{cases}$$

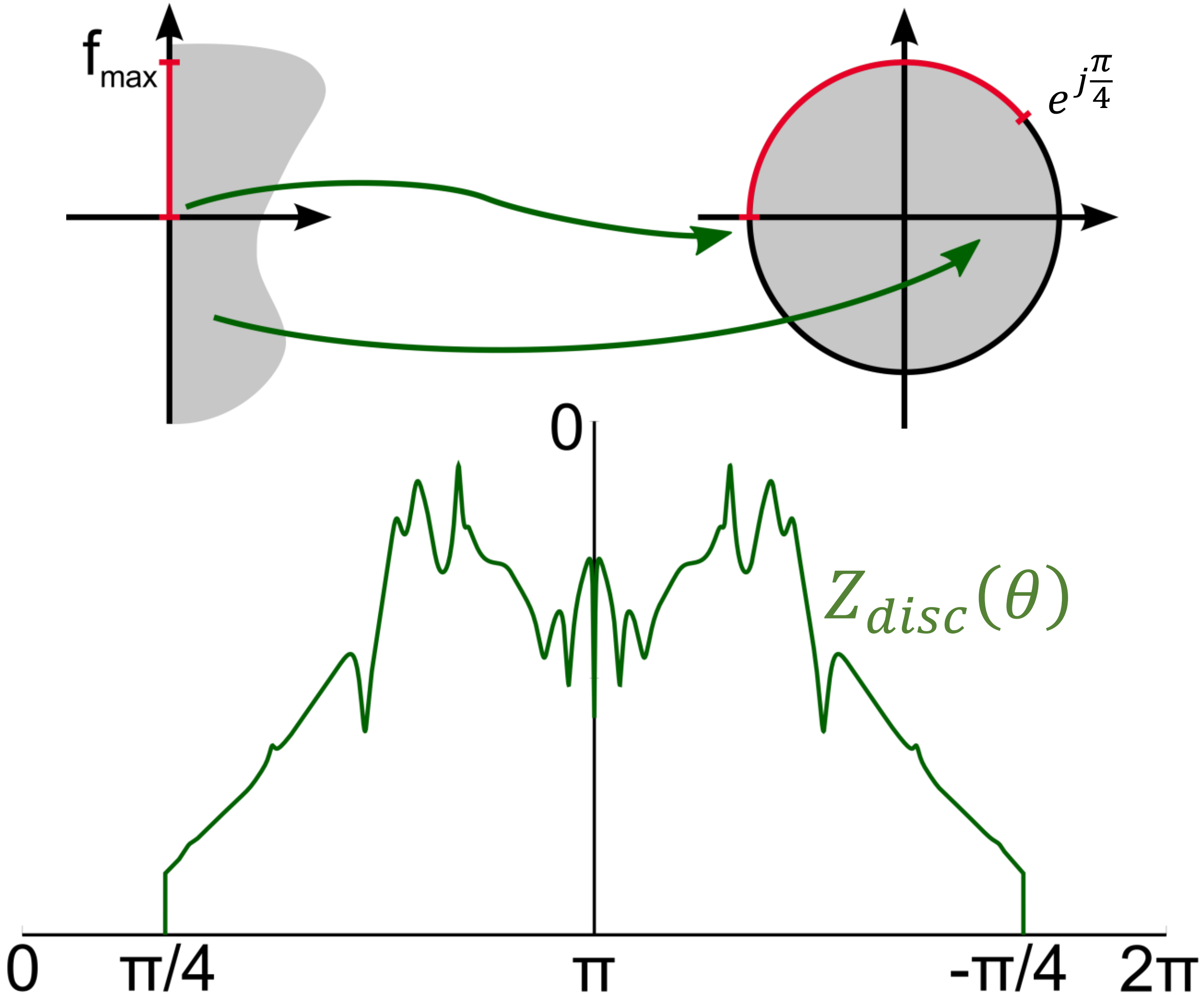
$$\mathcal{L}_2 \quad = \quad \mathcal{H}_2 \quad \oplus \quad \overline{\mathcal{H}_2}$$

$$Z(j\omega) = Z_{stable}(j\omega) + Z_{unstable}(j\omega)$$

$$Z_{stable}(j\omega) = P_{\mathcal{H}_2}\{Z(j\omega)\}$$

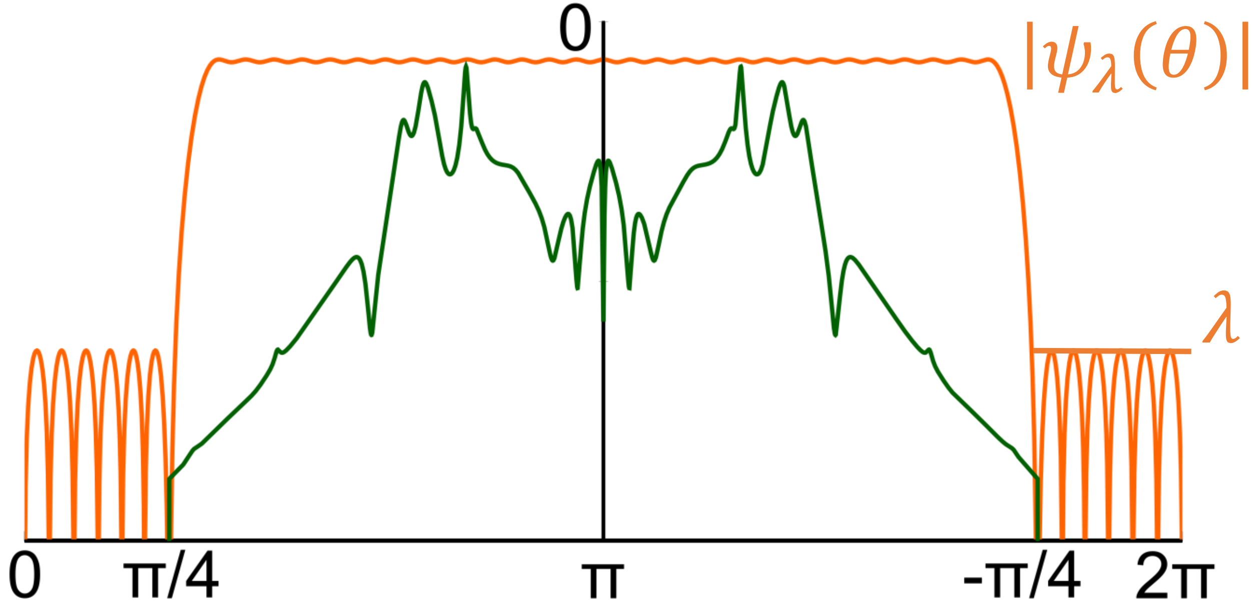
$$Z_{unstable}(j\omega) = P_{\overline{\mathcal{H}_2}}\{Z(j\omega)\}$$

Step 1: Transform to unit circle



Step 2: Multiply by Filter function

$$Z_f = Z_{disc} \cdot \psi_\lambda$$

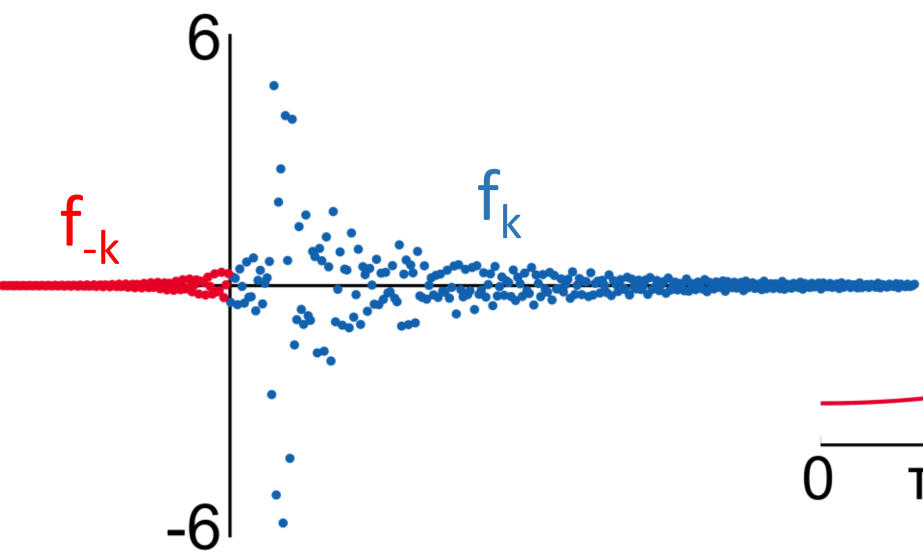


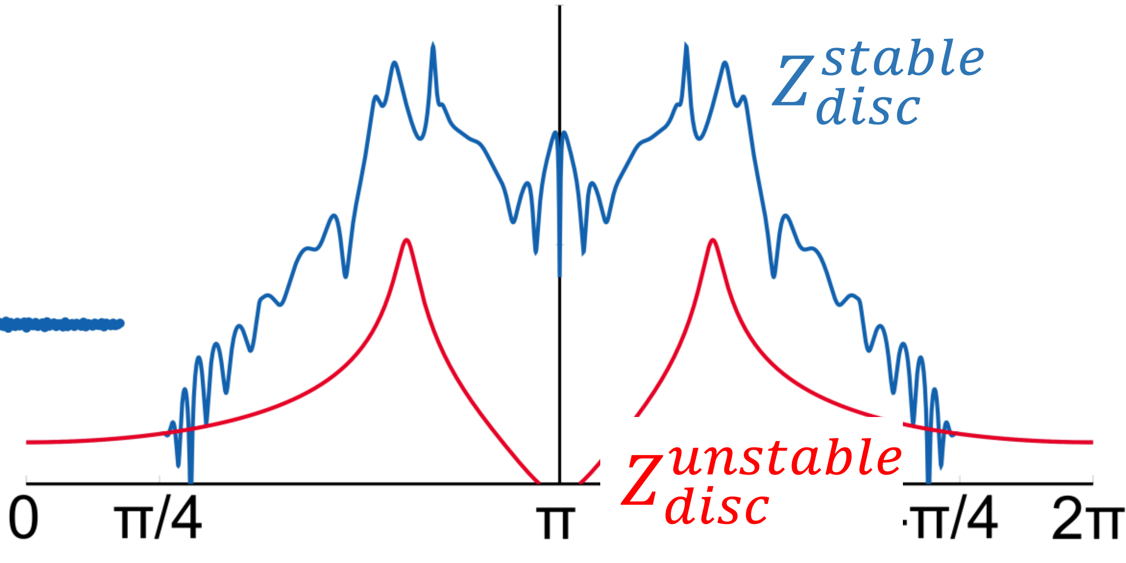
To smooth out edges

To suppress influence of out-of-band data

Step 3: Compute Fourier series

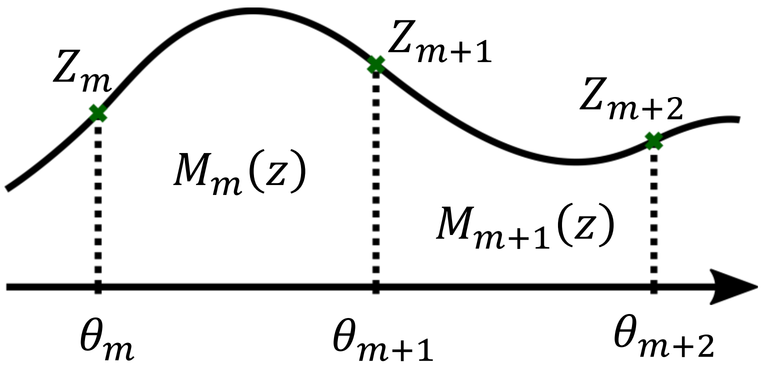
$$Z_f(\theta) = \underbrace{\sum_{k=0}^{\infty} f_k e^{jk\theta}}_{Z_{disc}^{stable}} + \underbrace{\sum_{k=1}^{\infty} f_{-k} e^{-jk\theta}}_{Z_{disc}^{unstable}}$$





Step 3: Compute Fourier series

Rational Interpolation



$$M_m(z) = \frac{az^2 + bz + c}{z + d}$$

Continuous derivative in interpolation points

Quadrature integration

$$f_k = \frac{1}{2\pi} \int_0^{2\pi} Z(\theta)\, e^{-jk\theta} d\theta$$

+ no extra parameters
- slow

Fast Fourier Transform (FFT)

+ Fast
- #points is extra parameter
- Introduces aliasing

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Stability Analysis by projection

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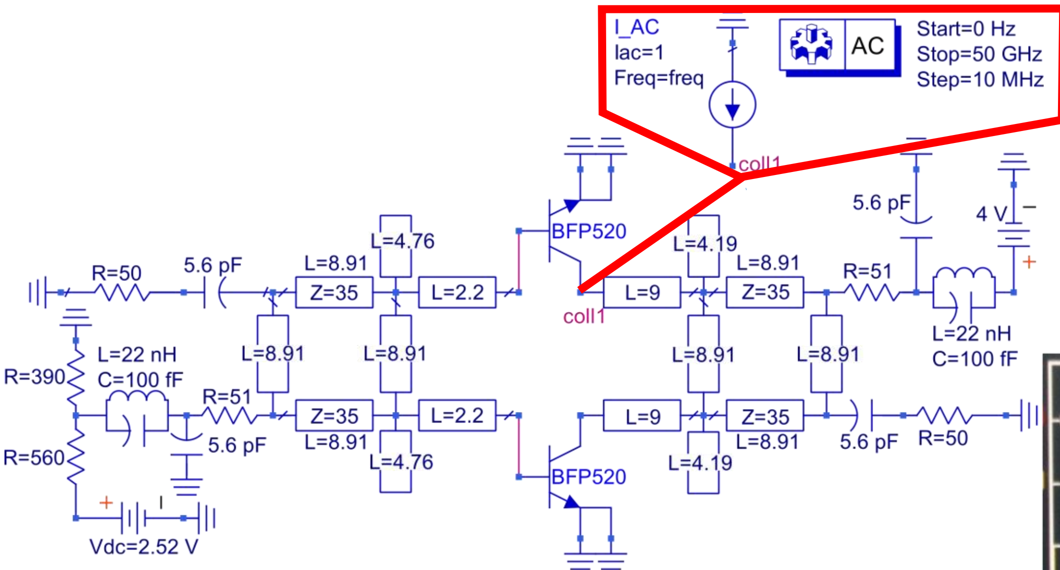
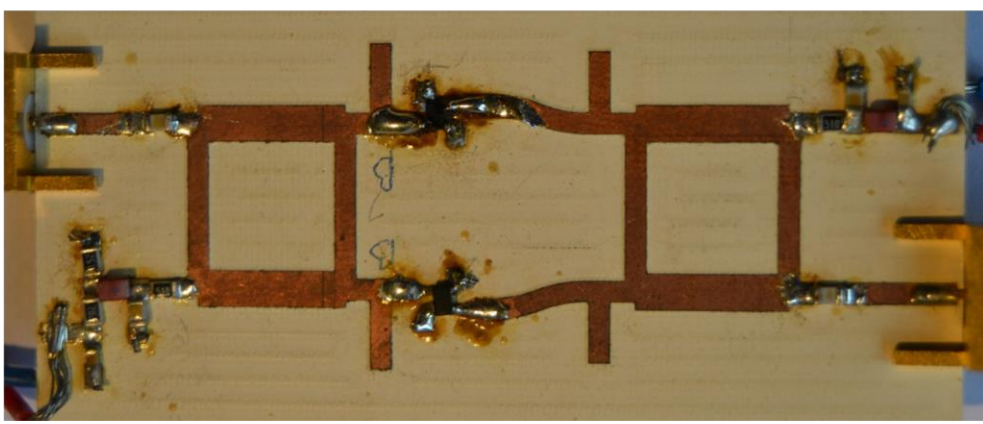
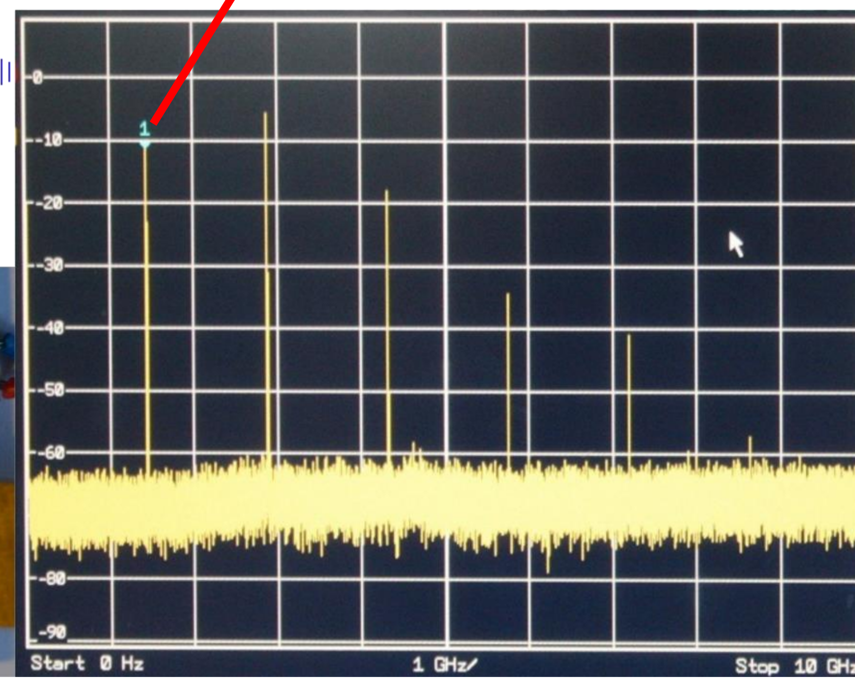
Example 1: Balanced Amplifier



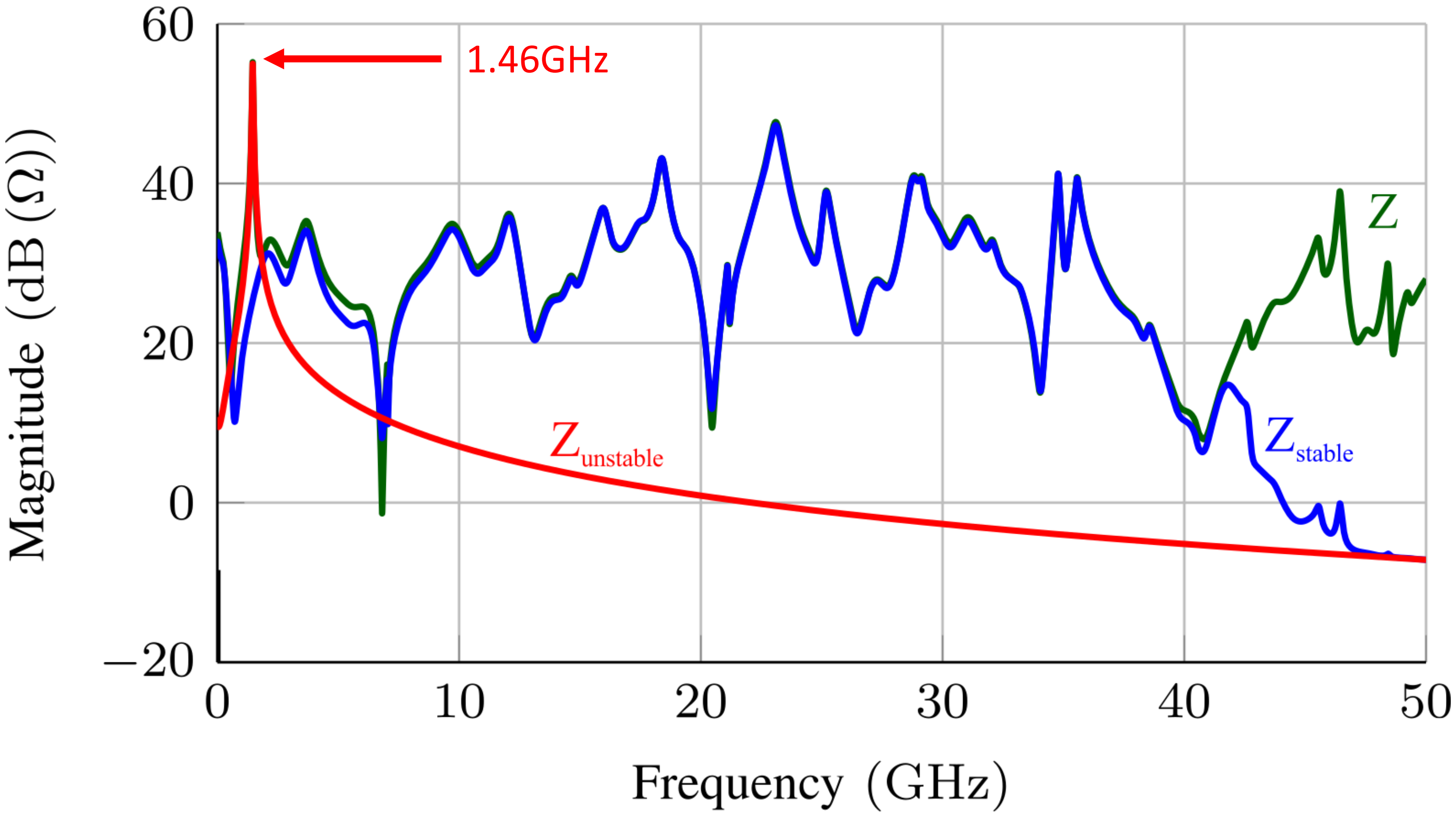


Instability measured at 1.43GHz



Result: Instability is detected

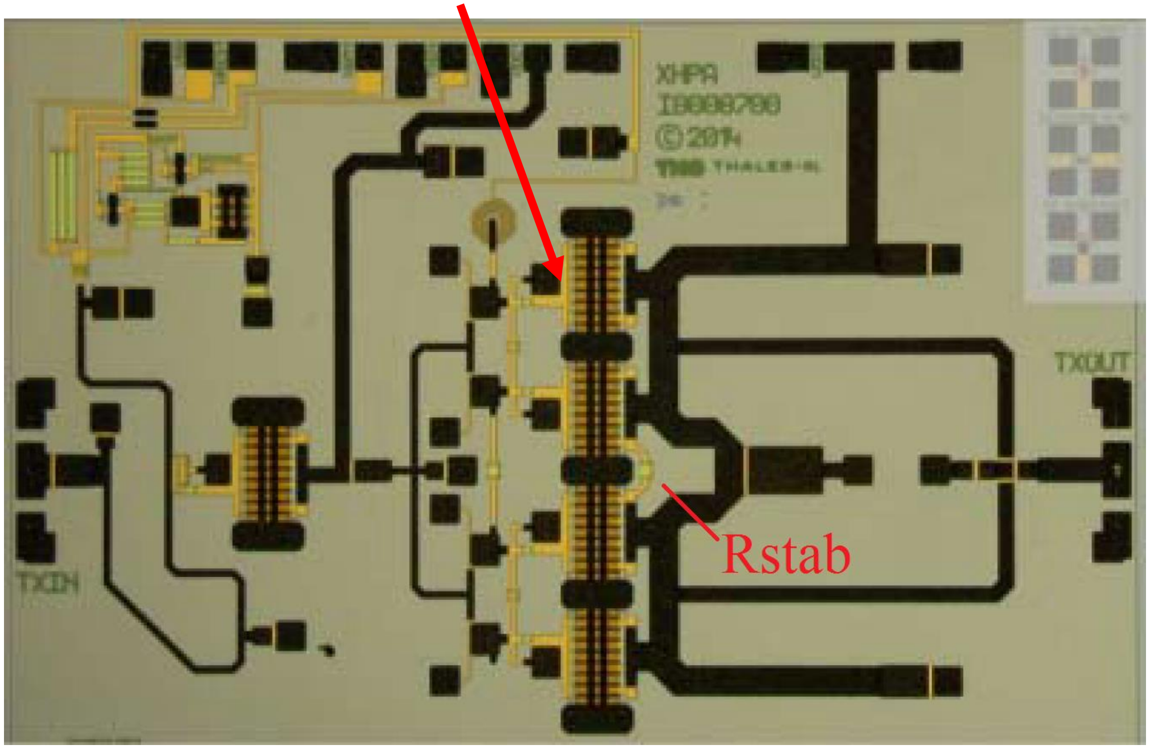
0 – 50GHz in 1MHz steps   50k points.   Processing time: 60ms



Example: Power Amplifier

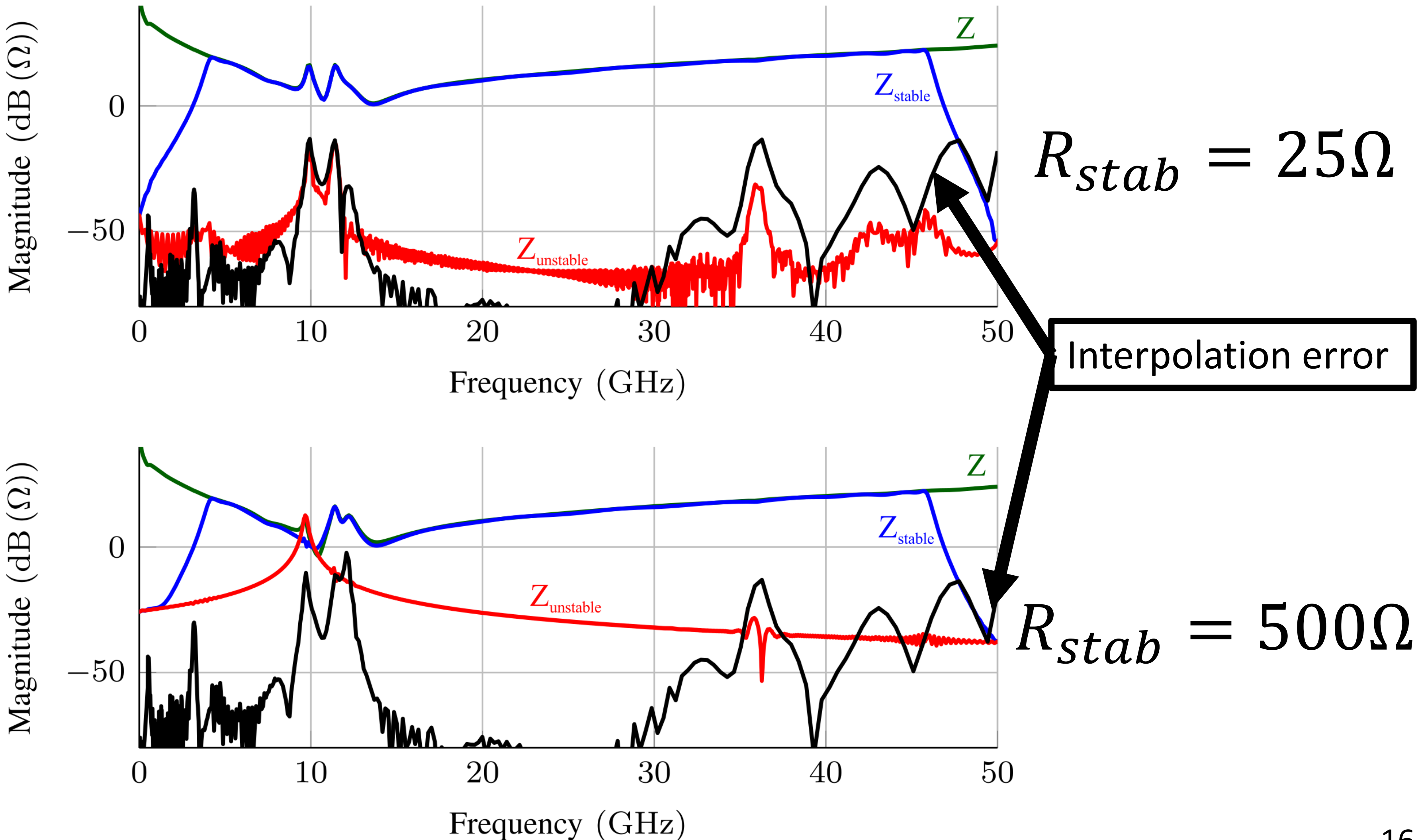
Possible odd-mode instability in second stage

Impedance determined here



Thanks to M. Van Heijningen (TNO) for the simulation data

Results



$R_{stab} = 25\Omega$

$R_{stab} = 500\Omega$

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Stability Analysis by projection

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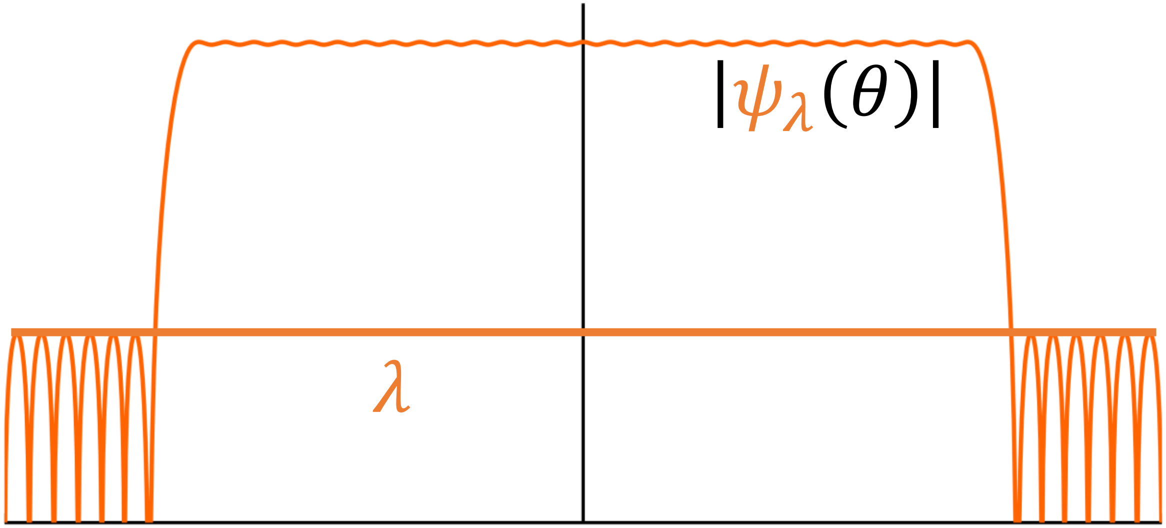
Influence of the filter



$Z(j\omega)$ unstable pole in γ_i:

$$P_{\bar{\mathcal{H}}_2}\{Z(j\omega)\} = \frac{R_i}{j\omega - \gamma_i} \Rightarrow P_{\bar{\mathcal{H}}_2}\{Z\psi_\lambda\} = \frac{\psi_\lambda(\gamma_i)R_i}{j\omega - \gamma_i}$$

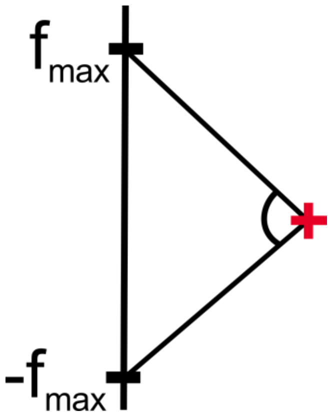
λ too low: might suppress poles

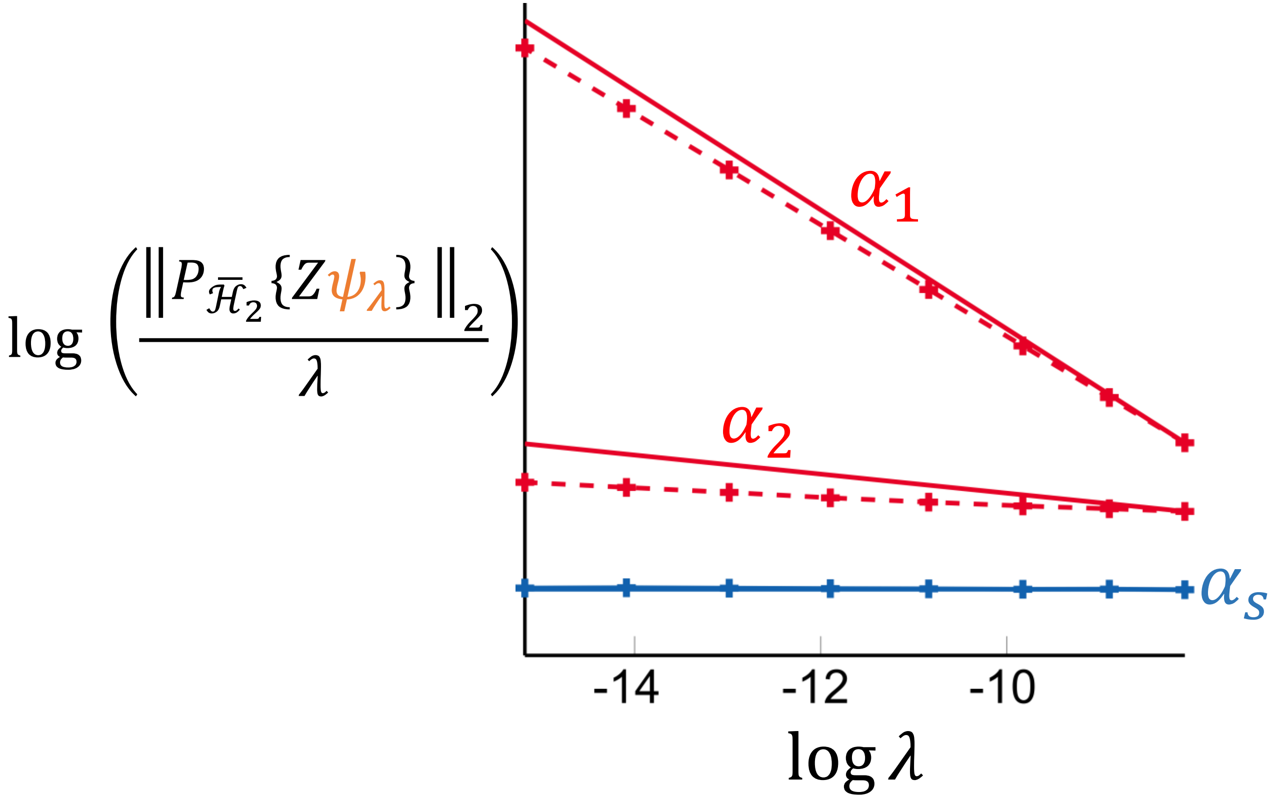
λ too high: influence of edge

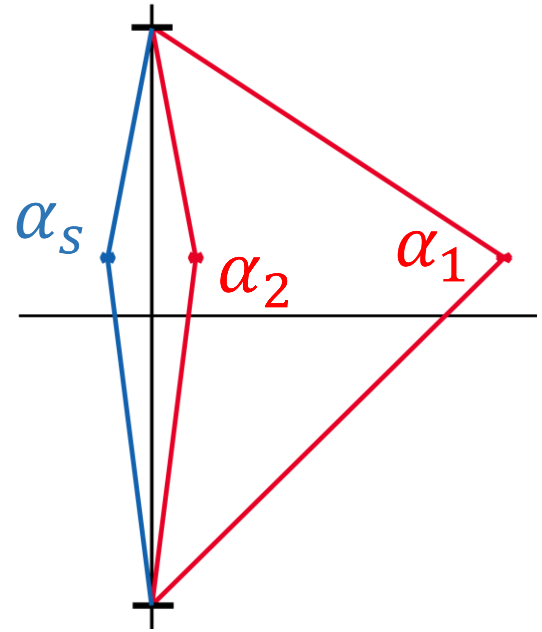
Projection as function of λ

Filter magnitude is known:

$$|\psi_\lambda(\gamma_i)| \cong \lambda^{1-\frac{\alpha}{\pi}}$$







Is $Z(j\omega)$ unstable?

Yes or ~~No~~

α very small

Stable OR Unstable pole far away

Projection + λ-analysis:

promising technique (Work in progress)

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Estimating unstable poles

Estimating unstable poles

Unstable part = rational

⇒ Classical methods to estimate the poles

Least squares

Levy's method, or more advanced

H_∞ approximation

Adamjan, Arov and Krein (AAK)

→ Padé approximation

Padé approx. of unstable part

When system has N poles in the unit circle then

$$\Psi_{N+1} = \begin{pmatrix} f_{-1} & f_{-2} & \cdots & f_{-(N+1)} \\ f_{-2} & f_{-3} & \cdots & f_{-(N+2)} \\ \vdots & \vdots & \ddots & \vdots \\ f_{-(N+1)} & f_{-(N+2)} & \cdots & f_{-(2N+1)} \end{pmatrix}$$

Has rank N

Use singular values of Ψ_M with $M > N$ to determine order

Padé approx. of unstable part

With order N known, compute SVD

$$\Psi_{N+1} = USW'$$

Poles of unstable part are now roots of

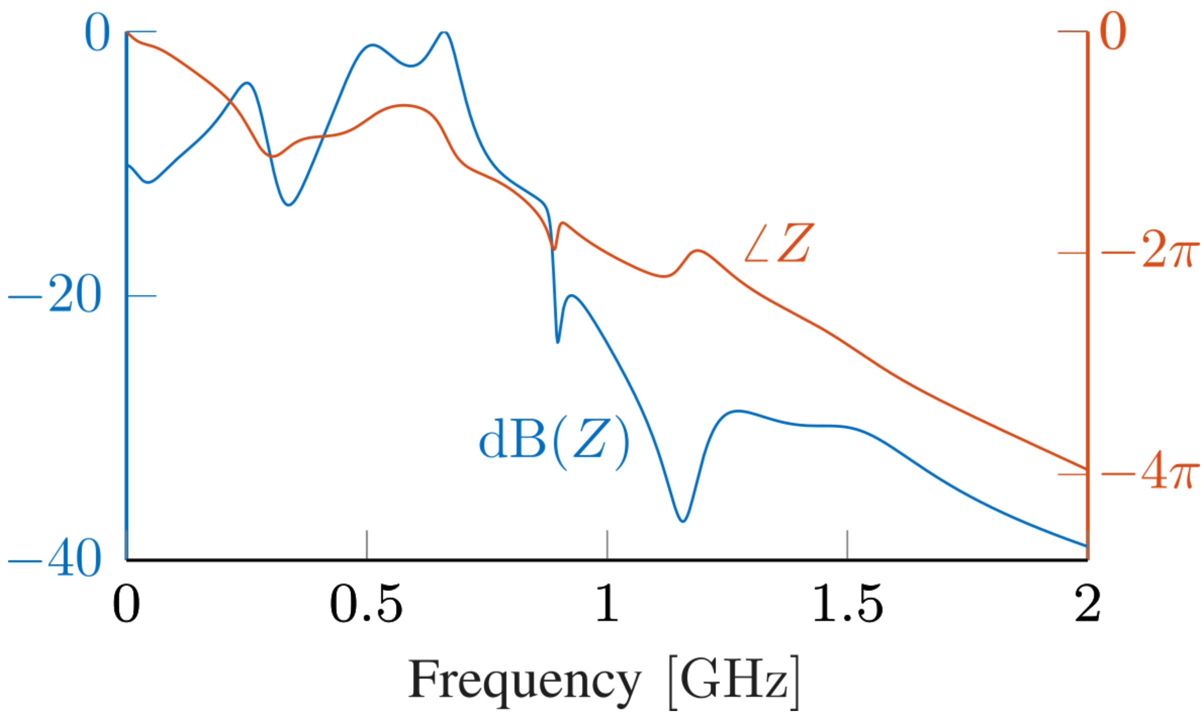
$$W_{N+1,N+1}z^N + W_{N,N+1}z^{N-1} + \cdots + W_{1,N+1}$$

Example: random system

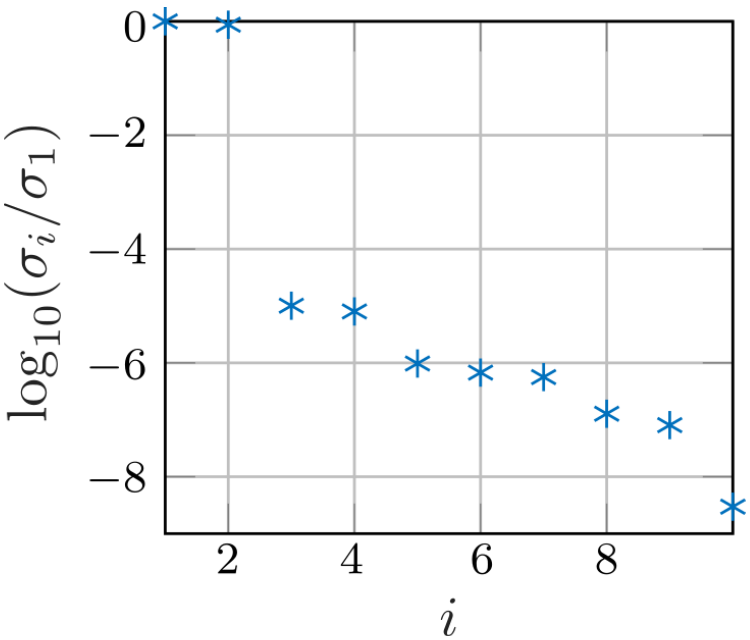
System with 52 poles and 50 zeroes. (RSS Matlab)

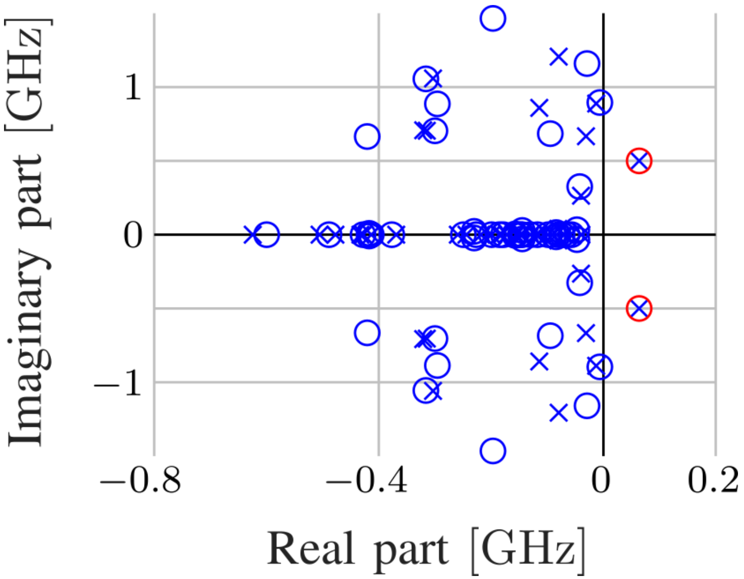
Time delay at the input (1ns)

2 unstable poles



Example: random system





Error = $\frac{|p_{estimated} - p_{correct}|}{|p_{correct}|} = 4.9 \cdot 10^{-6}$

Conlcusions

Stability analysis with projection

- “Model-free” method
- Allows non-parametric stab-analysis

Filter influence

- Could be used to get automatic yes/maybe answer

Determining unstable poles

- Exploit fact that unstable part is rational
- Padé approx. only requires small # Fourier coeffs