Functional Approach to stability analysis of linear systems

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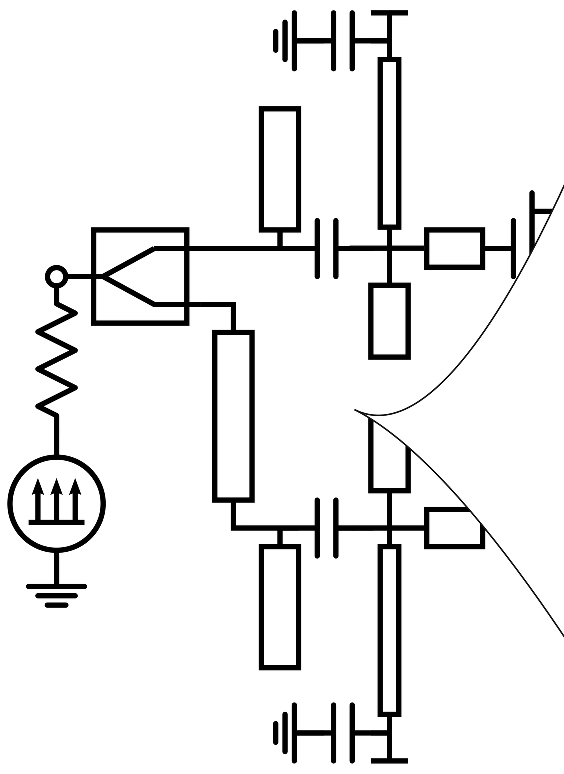


Frequency domain simulation methods

KEYSIGHT TECHNOLOGIES ADS

NATIONAL INSTRUMENTS MWO

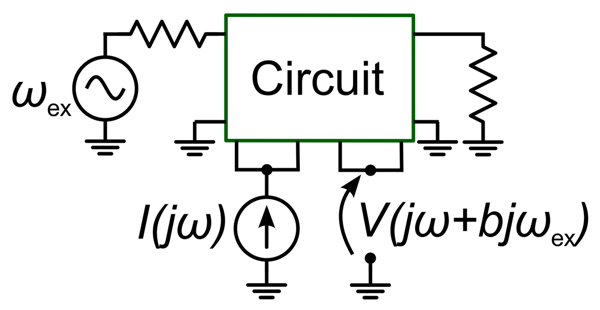
cadence Spectre

Proprietary simulator
Proprietary models

Equilibrium solution
DC simulation

Periodic solution
Harmonic Balance

Is the Solution Stable?
Linearise!



Result: Non-parametric Frequency response

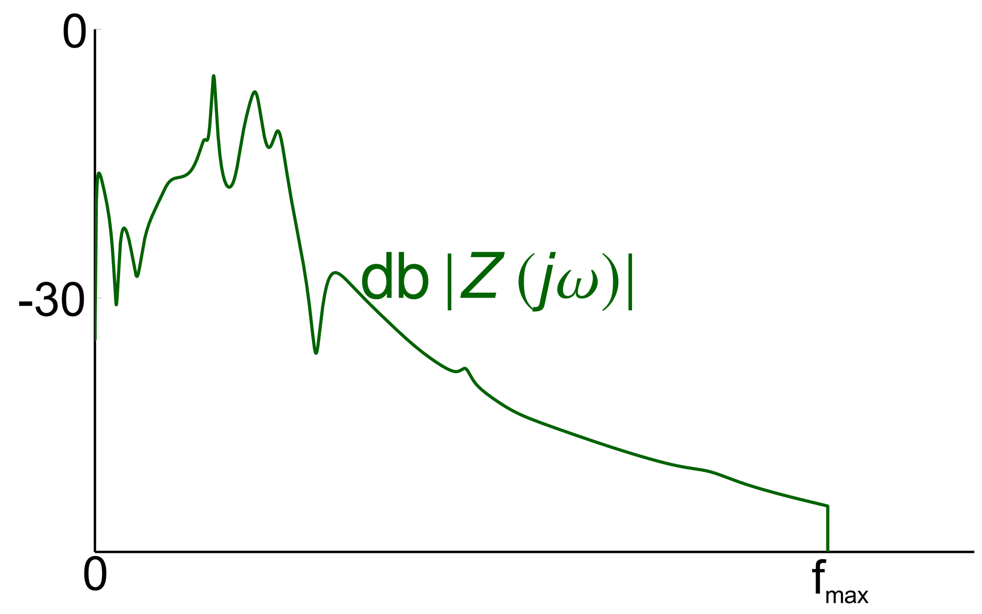
Linear Time Invariant system
AC simulation

$$Z(j\omega) = \frac{V(j\omega)}{I(j\omega)}$$

Linear Time-Periodic system
LSSS simulation

$$Z_b(j\omega) = \frac{V(j\omega + bj\omega_{ex})}{I(j\omega)}$$

Is a given frequency response stable or not?



Circuit contains delay $\Rightarrow Z(j\omega)$ is non-rational

Circuit is realistic $\Rightarrow$ finite number of unstable poles

Assume: unstable poles observable in $Z(j\omega)$

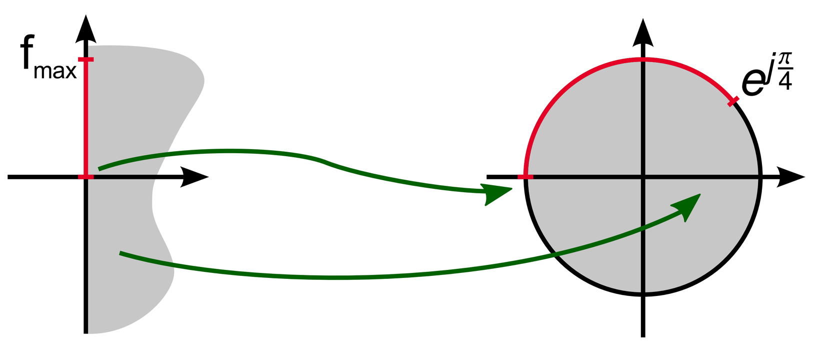
$Z(j\omega) \in L_2$ $\Rightarrow$ no poles on the axis

$$L_2 = \underbrace{H_2}_{\text{stable}} \oplus \underbrace{\overline{H_2}}_{\text{unstable}}$$

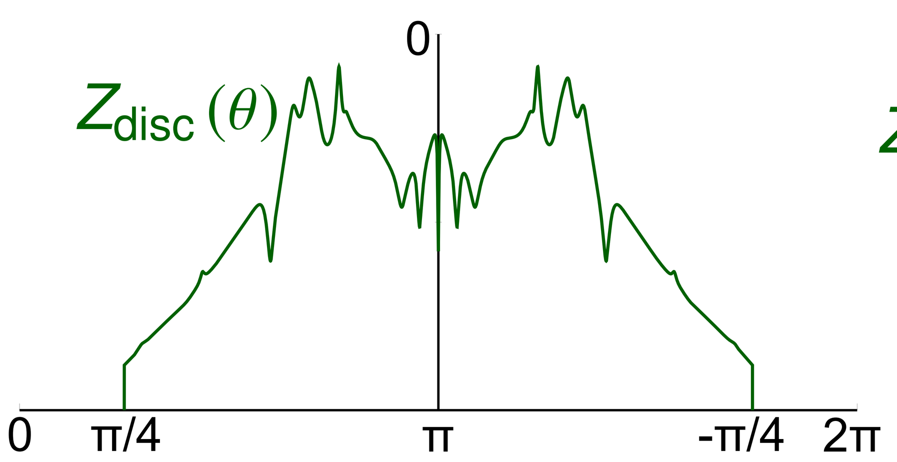
Check stability with projection

$$P_{\overline{H_2}}\{Z(j\omega)\}$$

1. Transform to unit circle

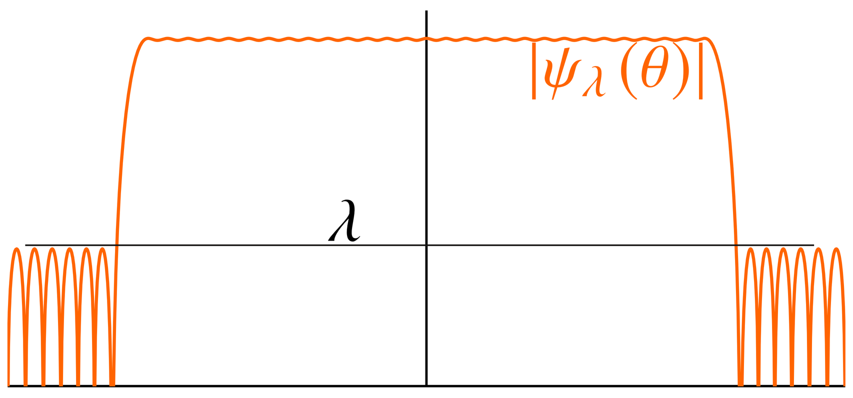


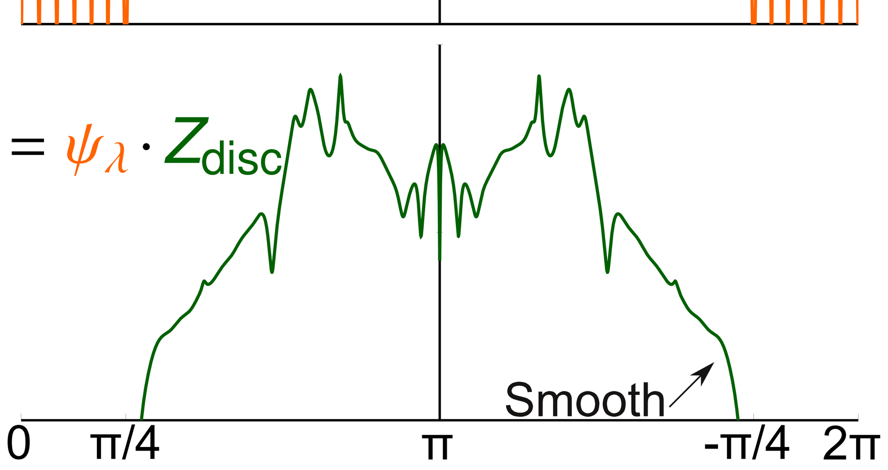
Bilinear/Mobius transform



2. Multiply by Filtering function

FIR filter (stable)
No zeroes on the disc

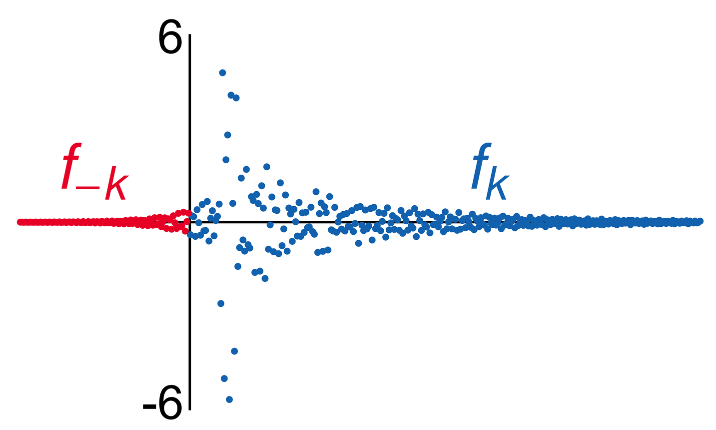




3. Compute Fourier series

$$Z_f(\theta) = \underbrace{\sum_{k=0}^{\infty} f_k e^{jk\theta}}_{Z^{stable}_{disc}} + \underbrace{\sum_{k=1}^{\infty} f_{-k} e^{-jk\theta}}_{Z^{unstable}_{disc}}$$

Interpolation + FFT for speed

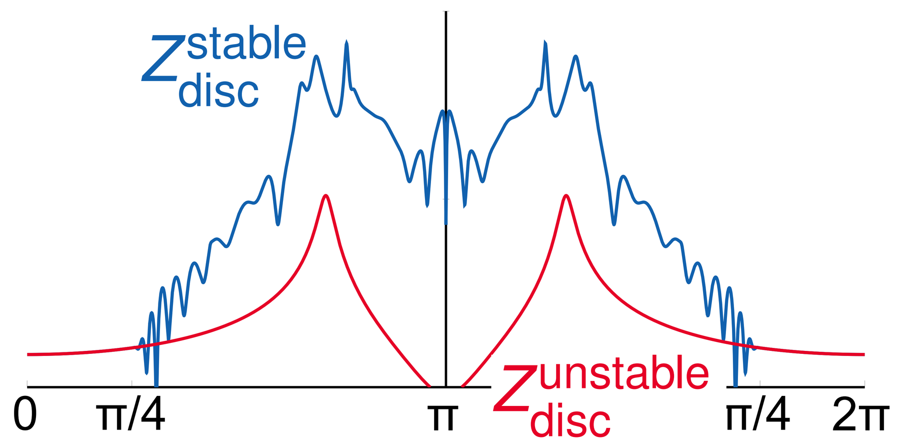


4. Estimate Unstable poles

Unstable part = rational

Use classical methods to recover the poles

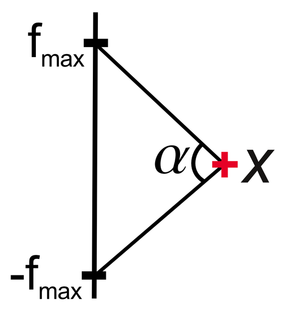
$$\text{svd}\left(\begin{bmatrix} f_{-1} & f_{-2} & f_{-3} & \cdots \\ f_{-2} & f_{-3} & \cdots & \cdots \\ f_{-3} & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix}\right)$$

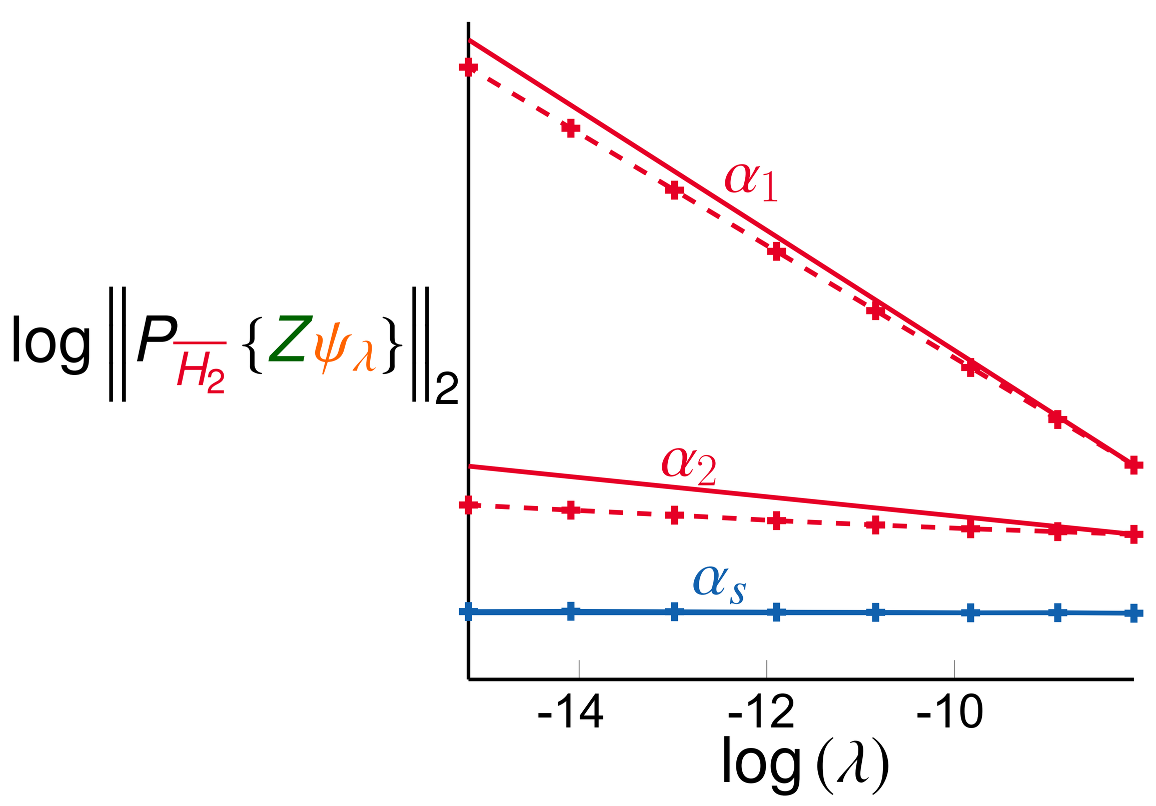


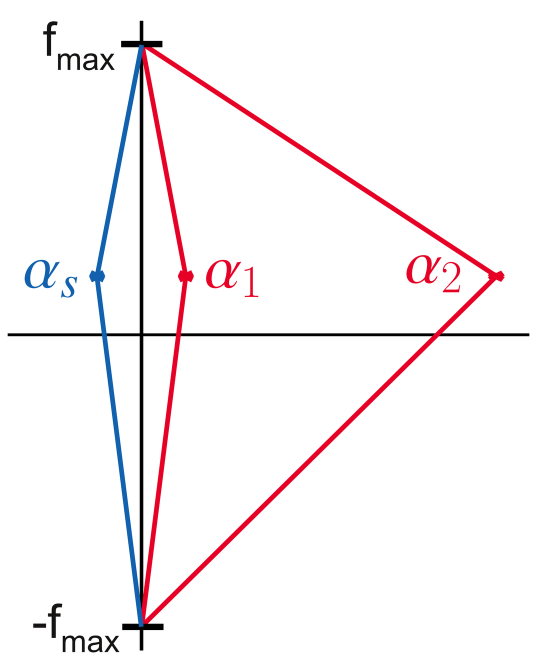
Influence of the filtering function

$$P_{\overline{H_2}}\{Z(j\omega)\} = \frac{\beta}{j\omega - x} \xrightarrow{\text{Filter}} P_{\overline{H_2}}\{Z(j\omega)\psi_\lambda\} = \psi_\lambda(x)\frac{\beta}{j\omega - x}$$

Filter magnitude is known: $|\psi_\lambda(x)| \cong \lambda^{1-\frac{\alpha}{\pi}}$







Conclusions

Determining stability? Project $Z(j\omega)$ onto $\overline{H_2}$

Finite interval? Use filtering function ψ_λ

Is $Z(j\omega)$ unstable? YES or ~~NO~~ $\rightarrow \alpha$ small

Stable OR Unstable pole far away

Future work

Pole close to axis $\Rightarrow$ large amount of Fourier coeffs needed

Change basis to cope with lowly damped poles

Analysis of periodic solutions

Copies of Floquet multipliers make $P_{\overline{H_2}}\{Z\}$ infinite order

Tracking critical stable poles?

Stable part ($P_{H_2}\{Z\}$) is non-rational

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APICS