Distortion Contribution Analysis of strongly non-linear analog circuits

Adam Cooman, Piet Bronders and Gerd Vandersteen

***Abstract*—A Distortion Contribution Analysis (DCA) determines the contributions of each sub-circuit to the total distortion generated by an electronic circuit in a simulation. The results of the DCA allow the designer of the circuit to effectively reduce the distortion.**

Recently, a DCA based on the Best Linear Approximation (BLA) was introduced. In this approach, the non-linear sub-circuits are modelled using a linear approximation. The non-linear distortion is represented as an additive noise source. Combining the BLA with the concepts of a noise analysis yields a DCA that works with realistic, modulated excitation signals instead of a one or two-tone excitation.

Up till now, BLA-based DCA has only been applied to weakly non-linear circuits. In this paper, it is extended and applied to a strongly non-linear circuit.

***Index Terms*—Distortion Contribution Analysis, Best Linear Approximation**

NON-LINEAR distortion has become an important limiting factor in obtaining a large dynamic range in analog electronic circuits. This has led to the need for circuit analyses in simulations to pinpoint the main sources of distortion, similarly to a noise analysis. The concept behind this Distortion Contribution Analysis (DCA), which determines the contributions of the sub-circuits to the total distortion generated by a circuit, was introduced long ago [1], [2].

Classical implementations of the DCA rely on the Volterra description of non-linear circuits [1], [2]. The Volterra-based approach introduces a very large number of contributions for large circuits or realistic excitation signals. Therefore, the analysis is always limited to simple excitation signals such as 1 or 2-tone excitations. To get a correct view on the distortion generated by the circuit however, the correct, modulated, excitation signal should be used [3].

Furthermore, the circuits are assumed to be weakly non-linear, which implies that every deviation from the small-signal behaviour of the circuit is considered non-linear distortion. This assumption renders the Volterra-based DCA useless when circuits are biased in class B or when switching elements are present. An elegant solution to this problem was proposed in [4] where only large-signal simulation results are used during the DCA.

More recently, a DCA based on the Best Linear Approximation (BLA) has been introduced [5]–[7]. Instead of using a full non-linear description of the sub-circuits, a linear approximation of the circuit behaviour is used. Combining the linear approximation with a classic noise analysis [8] yields a different kind of DCA [7] with some distinct advantages over Volterra-based DCA implementations, at the cost of a larger simulation time:

- Modulated signals are used in the analysis, giving a realistic view of the distortion generated in the circuit [3].
- The analysis of the circuit does not require access to the internal models of the devices and no specialised simulations are required.
- The DCA combines linear models and noise analysis methods with which most designers are already familiar.

Theoretically, the BLA can be used to model strongly non-linear circuits, but the BLA-based DCA has, until now, only been applied to weakly non-linear circuits [7]. For a weakly non-linear circuit, the underlying linear behaviour of the sub-circuit can be used to represent the BLA. The linear behaviour is easily obtained using small-signal simulations. Obtaining the BLA of a strongly non-linear circuit is not so easy, because only large-signal simulations, like harmonic balance, can be used. In this paper, the simulations and estimation algorithm to obtain the BLA of the sub-circuits are updated to be able to deal with strongly non-linear sub-circuits.

First, the basics behind the BLA are discussed (Section I). Then, the estimation of the BLA for a strongly non-linear circuit is considered (Section II). Finally, the BLA-based DCA is applied to a push-pull op-amp as an example (Section III).

I. The Best Linear Approximation

Instead of dealing with a deterministic input signal, like a sine wave or a two-tone, the approach behind the BLA is to consider only the average behaviour of a non-linear system when random input signals are considered. Only the Power Spectral Density (PSD) and Probability Density Function (PDF) of the input signal $\mathbf{U}$ are fixed to resemble the signals the system will encounter in its application. [9] then shows for a large class of non-linear systems[1], that the relation between the input $\mathbf{U}$ and the output $\mathbf{Y}$ at a frequency f can be written as (Figure 1):

$$\mathbf{Y}(f) = \mathbf{G}_{\text{BLA}}(f)\,\mathbf{U}(f) + \mathbf{D}(f) \tag{1}$$

where $\mathbf{G}_{BLA}(f)$ is the Best Linear Approximation in least squares sense, defined as

$$\mathbf{G}_{\text{BLA}}(f) = \frac{S_{yu}(f)}{S_{uu}(f)} \tag{2}$$

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[1] The class of Periodic-In Same Period Out or PISPO systems are non-linear systems that can contain strongly non-linear elements like a saturation or a discontinuity, but rule out hysteretic and chaotic systems.

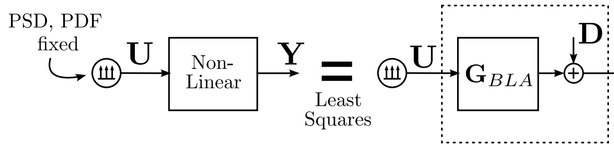

Figure 1. The response of a non-linear system to a fixed class of input signals can be approximated by a linear response. The residual is the distortion introduced by the system.

with S_{yu} being the input-output cross-power spectrum and S_{uu} the input autopower spectrum. The residual $\mathbf{D}$ in (1) is the non-linear distortion introduced by the system. Due to the choice of $\mathbf{G}_{\text{BLA}}$, $\mathbf{D}$ has the properties of a noise source: it is zero-mean and uncorrelated with the input signal $\mathbf{U}$ [9].

Using the framework of the BLA has some advantages when analysing the non-linear behaviour of a circuit:

- The user is forced to use the same class of signals that the circuit will encounter in its application. These realistic input signals will lead to an accurate representation of the distortion encountered in the system
- When the input power goes to zero, $\mathbf{G}_{\text{BLA}}$ converges to the small-signal behaviour of the circuit.
- Interpretation of the results is relatively easy, because it only requires knowledge of linear systems theory and noise analysis.

In order to determine the BLA, the circuit has to be excited with several realisations from the specified signal class. Averaging the circuit response over the realisations allows extraction of the BLA. In practice, random-phase multisines (RPM) are used. A RPM is a sum of harmonically related sines with a specified amplitude spectrum, but a random phase:

$$u(t) = \sum_{k=1}^{N} A_k \sin\left(2\pi k f_0 t + \phi_k\right) \qquad (3)$$

A_k and ϕ_k are the amplitude and phase of the k^{th} harmonic of the multisine. f_0 is the base frequency of the RPM. The A_k are fixed to obtain the correct PSD, while the ϕ_k are chosen from a uniform distribution $[0, 2\pi[$ to obtain a Gaussian PDF. The BLA obtained with a RPM is the same as for a filtered Gaussian noise source with the same PSD if the amount of tones N in the RPM goes to infinity [9]. The benefit of working with a periodic signal like the RPM is that common circuit simulation techniques, like harmonic balance, can be used to obtain the steady-state response of the circuit. These steady-state responses can then be used to determine the BLA.

A. Example 1: BLA of a class-C amplifier

Consider a simple class-C amplifier designed in a commercial $0.18\mu\text{m}$ technology (Figure 2). The circuit is excited with a random-odd RPM with a flat PSD between 1Hz and 100Hz. An odd RPM has $A_k = 0$ for all even k. On top of that, one random tone is left out of each group of four odd tones to obtain a random-odd RPM. With this excitation signal, the even and odd non-linear distortion, generated by the circuit,

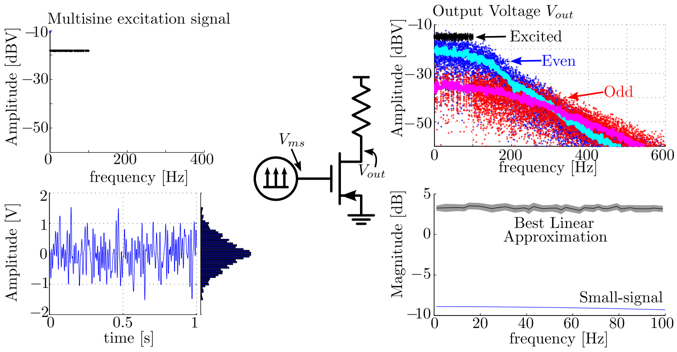

Figure 2. The BLA gives intuitive insight into the working of this CMOS Class-C amplifier. The input multisine is shown on the left in both frequency and time domain. The output spectrum obtained with the multisine excitation is shown on the top right. ● are the odd frequency bins excited by the multisine, ● are the even bins and ● are the remaining odd bins. ● and ● indicate the rms power of the distortion sources in the circuit. The BLA estimate and its uncertainty are shown in black on the bottom right. The small-signal behaviour is shown in blue.

can be separated by just considering the result on the even and odd frequency lines respectively.

The root-mean square voltage of the RPM is 0.5V and its PDF is close to Gaussian (shown on the right in Figure 2). The steady-state response of the circuit to the RPM is obtained with a commercial harmonic balance simulator (Keysight's Advanced Design System). 30 different phase realisations of the input multisine are simulated.

The spectrum of the output voltage for each multisine is shown on the top right in Figure 2. The noisy behaviour of the non-linear distortion is seen clearly. The even and odd non-linear distortions are separated because an odd multisine is used.

By considering the average frequency response at the excited bins (shown in black), the BLA is obtained (bottom right in Figure 2). Because the circuit is biased in class C, the BLA deviates strongly from the small-signal behaviour of the transistor.

II. Estimating the BLA of a sub-circuit

In a Distortion Contribution Analysis based on the BLA, a standard noise analysis is applied to the distortion sources in the circuit [7]. The noise analysis [8] works with the S-parameters of each of the sub-circuits in the circuit under test[2]. The BLA-based DCA will therefore require Multiple-Input Multiple-Output (MIMO) BLAs that relate the input and output waves of each sub-circuit. For a sub-circuit with two ports, the following expression is obtained for the BLA of the sub-circuit at a certain frequency:

$$\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} S_{11}^{\text{BLA}} & S_{12}^{\text{BLA}} \\ S_{21}^{\text{BLA}} & S_{22}^{\text{BLA}} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} + \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} \qquad (4)$$

[2]Waves and S-parameters are used in our implementation of the DCA because the S-parameters of degenerate circuits like an open, short-circuit or a through connection exist, whereas Y or Z parameters can become infinite or zero in those cases.

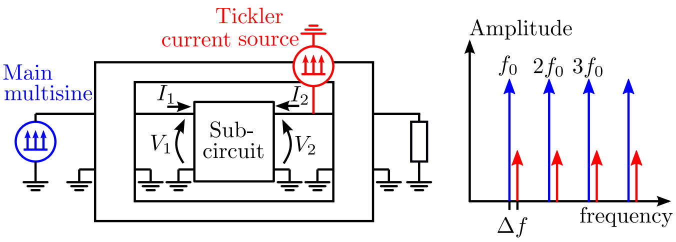

Figure 3. Estimating the BLA of a sub-circuit requires extra tickler multisines (red) around the sub-circuit. Placing the ticklers on a frequency grid with a small offset Δf allows to obtain the response of the tickler without influence from the main multisine.

The A_i and B_i waves are related to the voltages and currents at the same frequency in the classical way:

$$A_i = \frac{V_i + Z_0 I_i}{2\sqrt{Z_0}} \qquad B_i = \frac{V_i - Z_0 I_i}{2\sqrt{Z_0}} \tag{5}$$

with Z_0 a chosen reference impedance. The distortion sources D_1 and D_2 in (4) are wave sources. They are zero-mean and uncorrelated to the input multisine and are described by the covariance matrix $\mathbf{C}_D$:

$$\mathbf{C}_D = \mathbb{E}\left(\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}^{\mathsf{H}} \right) \tag{6}$$

where $\mathbb{E}$ indicates the expected value and $^{\mathsf{H}}$ indicates the hermitian transpose.

Estimating the MIMO BLA in (4) requires at least as many experiments as there are inputs to the circuit. The input-output vectors of the different experiments are stacked next to each-other to obtain full matrices and the BLA can then be estimated as follows:

$$\begin{bmatrix} S_{11}^{\text{BLA}} & S_{12}^{\text{BLA}} \\ S_{21}^{\text{BLA}} & S_{22}^{\text{BLA}} \end{bmatrix} = \begin{bmatrix} B_1^{[1]} & B_1^{[2]} \\ B_2^{[1]} & B_2^{[2]} \end{bmatrix} \left(\begin{bmatrix} A_1^{[1]} & A_1^{[2]} \\ A_2^{[1]} & A_2^{[2]} \end{bmatrix} \right)^{-1} \tag{7}$$

To obtain a well-conditioned input matrix in this expression, a second RPM source, called the tickler, is added to the circuit (Figure 3). Very low amplitude current sources are used to avoid disturbing the non-linear operating point set by the main RPM. When the tickler and the main multisine share the same fundamental frequency f_0, the response to the tickler is overwhelmed by the non-linear distortion generated by the circuit. Adding a small frequency offset Δf to the tickler base frequency solves this problem at the cost of a longer simulation time. This technique is called 'zippering' the multisines [10].

When working with zippered multisines, first, the BLA from each multisine source to the inputs and outputs of the sub-circuit is calculated at the frequency grid determined by the corresponding multisine. Then, the obtained frequency responses are interpolated onto the wanted frequency grid. Finally, the BLA of the sub-circuit is calculated by performing (7). This indirect method leads to an unbiased estimate of the BLA, as shown in [11].

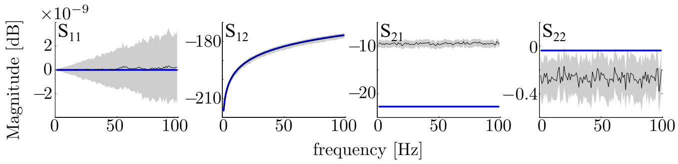

Figure 4. Using an extra tickler multisine at the output of the class-C amplifier allows to determine the MIMO BLA of the transistor. The BLA and its 3σ uncertainty bound are shown in black. The small-signal S-parameters are shown in blue.

Example 2: MIMO BLA of the class-C amplifier

Recall the class-C amplifier from example 1, but now consider the 2×2 MIMO BLA of the transistor. The main multisine is left unchanged from the previous example. A RPM tickler current source with an amplitude of $1\mu\text{A}$ is added to the output of the transistor to be able to perform the MIMO estimation. The frequency grid of the tickler multisine is shifted 1mHz away from the main multisine. This can be done easily in a harmonic balance simulation by adding 1mHz as an additional fundamental frequency with an order of 1. In a transient or periodic steady-state simulation, the frequency offset should be set to $f_0/2$ to avoid an excessive simulation time.

The steady-state voltages and currents are measured during the simulation to allow calculation of the waves in Matlab. Again, 30 different phase realisations are simulated to obtain a good estimate of the BLA. The results are shown in Figure 4. Again, the BLA deviates strongly from the small-signal behaviour of the circuit due to the fact that it is biased in class C. The largest difference is observed on the forward gain S_{21} and the output reflection S_{22} of the amplifier, which is to be expected.

III. BLA-BASED DCA

The BLA of each sub-circuit can now be used in the BLA-based DCA without changing the original algorithm described in [7]. The only difference lies in the estimation of the BLA of each sub-circuit. In [7], an S-parameter simulation was used to determine the small-signal behaviour of the sub-circuit and that small-signal behaviour was used in the algorithm. This approach is justified by the fact that the circuit is weakly non-linear. For strongly non-linear circuits, the BLA deviates far from the small-signal behaviour, so the new identification algorithm with zippered multisines must be used.

An op-amp with a push-pull output stage is used as example. Its architecture is a GA-CF-GA configuration as is detailed in chapter 7.7 of [12] (Figure 5). A commercial $1.8\mu\text{m}$ CMOS technology with a 3.3V supply voltage was used. The op-amp is placed in an inverting feedback configuration with a gain of 5 and drives a load capacitance of 10pF and a load resistance of $1\text{k}\Omega$ with a gain-bandwidth product of 10MHz. The excitation signal is an odd multisine which excites frequencies up to 1MHz with a base frequency of 10kHz and with an amplitude of 80mV_{rms}. With a gain of 5, this will result in a Gaussian distributed output signal with

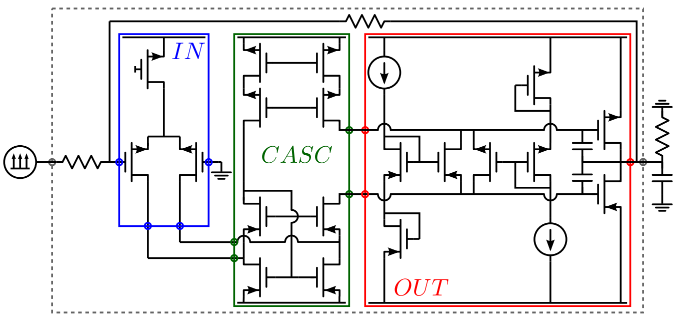

Figure 5. Push-pull op-amp used as an example. The op-amp is placed in an inverting feedback configuration with a gain of 5. The load consists of a 10pF capacitance and a 1kΩ resistor.

a rms voltage of $0.4\mathrm{V_{rms}}$. The op-amp is divided into three different stages as shown in Figure 5.

The cascode stage contains 4 ports, so three tickler multisines are added to set-up. All multisines are placed on a non-overlapping frequency grid. Estimation of the MIMO BLA of the sub-circuits in the op-amp is more difficult than for the class-C amplifier, because the reverse gain of the sub-circuits is very small. For both the cascode and output stages, the reverse gain had to be replaced by the one obtained with a small-signal simulation to obtain accurate results.

The DCA finds that the main source of non-linear distortion is due to interaction between the cascode stage and the output (Figure 6). To verify the results, the sum of all non-linear contributions is compared to the total distortion found at the output of the total circuit. A good match is obtained over the full frequency band (Figure 7b). When the small-signal behaviour of sub-circuits is used in the DCA, a difference of up to 15dB is noticed between the sum of contributions and the actual distortion in the output spectrum (Figure 7a), clearly indicating the need for the correct estimation of the BLA.

IV. Conclusion

In this paper it is shown that the BLA-based DCA can also be used on simulations of circuits that are not weakly non-linear. Examples include a class-C amplifier and an op-amp with a push-pull output stage. The main difference to previous implementations of the BLA-based DCA lies in the estimation method for the BLA of the sub-circuits. It is shown that the MIMO BLA of the strongly non-linear sub-circuits can be estimated by adding very small amplitude RPM current sources to the circuit and placing them on an interleaved frequency grid. The obtained BLA can then be used in the DCA without needing changes to the original DCA.

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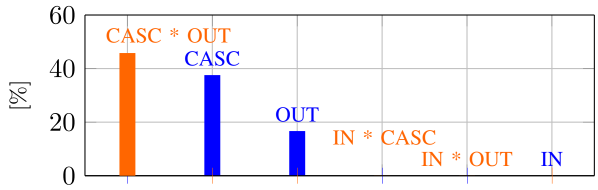

Figure 6. The DCA finds that the dominant source of distortion is the cascode stage. The strong correlation with the output stage (shown in orange) indicates that the distortion is generated at the interface between both stages. All other parts contribute for less than 1% to the total distortion.

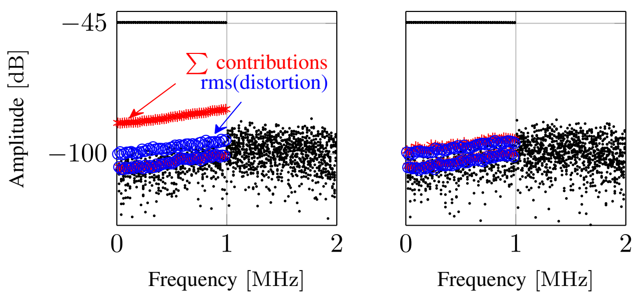

Figure 7. Using the small-signal behaviour of the circuits in the DCA leads to a large error (a). With the new estimation algorithm for the BLA, the error is removed (b). The steady-state spectra for the different realisations of the multisine are shown in black, The sum of the contributions (+) should match the rms of the circuit distortion (○).

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