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Wideband Distortion Contribution Analysis of Analog Circuits with Differential Signalling

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***Abstract*—Distortion Contribution Analysis (DCA) is a simulation-based analysis in which the total output distortion generated by an analog electronic circuit is split into the contributions of each of its sub-circuits. Earlier work has shown that combining the Best Linear Approximation (BLA) with a classic noise analysis yields a DCA which uses a wideband excitation signal without requiring knowledge about the underlying technology. In this paper, the BLA-based DCA is augmented to be able to cope with circuits that use differential signalling. Transforming the S-parameters of the sub-circuits in the DCA into a representation that works on the differential and common-mode signals leads to distortion contributions which are easy to interpret when differential signals are present in the circuit. Additionally, an improved test is introduced to indicate the accuracy of the DCA.**

The ever-increasing complexity of communication signals and the continuous lowering of the supply voltage increases the importance of the non-linear distortion generated by analog electronic circuits. During design of analog circuits however, modern simulators will only give an indication of the total distortion generated by a circuit, using, for example, intermodulation and intercept points. Finding where the origin of the distortion in the circuit is left to the intuition of the designer.

A noise analysis is available in commercial simulators for noise generated in circuits. This analysis gives the noise contribution of each element in the circuit to the total noise of the circuit [1]. A similar analysis that calculates the contributions of non-linear distortion to the total distortion of the circuit is called a Distortion Contribution Analysis (DCA).

Classically, a DCA is performed using Volterra Theory [2]. Implementing these techniques is complicated, due to the complexity of the Volterra models. To deal with this complexity the excitation signal is simplified single sines and reduced Volterra models are used in the analysis.

More recently, an alternative to the Volterra-based DCA has been proposed [3]. This alternative DCA uses wideband excitation signals and a specific linearisation of the sub-circuits called the Best Linear Approximation (BLA). The distortion contributions of each sub-block to the total distortion can be obtained when the BLA is combined with a classic noise analysis [4], [5]. Working with the BLA to perform the DCA has a few advantages over the Volterra-based approaches:

- A realistic excitation signal is used, which places the circuit in its correct non-linear operating point [6]
- The BLA-based DCA is very similar to a noise analysis, something most designers are already familiar with
- The approach does not require any knowledge of the underlying technology or circuitry

A major drawback of the method lies in the fact that determining the BLA of each sub-circuit requires a long simulation time. However, the BLA can be approximated by the small-signal behaviour of the circuit if the circuit operates close to its linear region. This approximation introduces a small error in the analysis, but saves simulation time [5]. The first contribution of this paper is to add a test to the DCA that estimates the error made in the approximation.

Modern analog electronic circuits regularly use differential signalling to become immune to disturbances from the power supply and to increase the available signal swing. Even basic building blocks in single-ended circuits contain differential signals. The differential pair at the input of an op-amp is a good example. Differential signalling has a large influence on the non-linear distortion as well: if the circuit is perfectly symmetric, even-order distortion will cancel out. A DCA needs to be able to handle differential signals in the circuits elegantly. If differential signal lines are left ungrouped, cancelling non-linear contributions will appear at the output of the circuit, possibly obscuring the important non-linear contributions. Because the BLA-based DCA is already using concepts from linear circuit theory, adding differential signalling can be done elegantly. This paper shows that introducing mixed-mode S-parameters [7] into the BLA-based DCA enables to handle the differential signals correctly.

The remainder of this paper is structured as follows: Section I summarises the basics behind the DCA based on the Best Linear Approximation. A new test to check the accuracy of the method is detailed in Section II. In Section III mixed-mode S-parameters are introduced into the method to handle differential signals in the circuit. Section IV applies the method to two different two-stage operational amplifiers: a Miller op-amp and a Folded-Cascode op-amp.

I. DCA using the Best Linear Approximation

A summary of the existing Distortion Contribution Analysis based on the Best Linear Approximation is provided in this section. The original algorithm is described in more detail in [5] while more details about the BLA can be found in [8].

A. *Excitation signal*

The excitation signal used in the DCA is a random-phase multisine, which is a sum of N harmonically related sines:

$$U(t) = \sum_{k=1}^{N} A_k \sin\left(2\pi k f_0 t + \phi_k\right) \quad (1)$$

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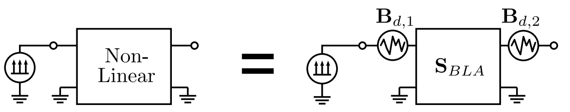


Fig. 1. The behaviour of a non-linear circuit can be approximated in least-squares sense by a linear circuit $\mathbf{S}_{BLA}$ and zero-mean distortion noise sources $\mathbf{B_d}$

where f_0 is the base frequency of the multisine, A_k is the amplitude of the k^{th} frequency line and ϕ_k is a random phase drawn from a uniform distribution $[0, 2\pi[$. This wideband excitation signal has a Gaussian probability density function (PDF). The amplitudes of the frequency lines should be chosen such that the power spectral density (PSD) of the multisine mimics the signals the circuit will encounter in its application.

B. Best Linear Approximation

If the behaviour of a non-linear circuit, excited by a signal with a fixed PSD and PDF is approximated in least squares sense by a linear system, then the distortion added by the circuit can be modelled by noise sources which are uncorrelated to the excitation signal [8]. For an n-port circuit, this means that the vector of n output waves $\mathbf{B}$ at a frequency f can be written as follows (Figure 1):[1]

$$\mathbf{B}(f) = \mathbf{S}_{BLA}(f)\mathbf{A}(f) + \mathbf{B}_d(f) \tag{2}$$

where $\mathbf{A}$ is the column vector of n incident waves to the circuit. $\mathbf{S}_{BLA}$ is the Best Linear Approximation: an $n \times n$ frequency response matrix of scattering parameters. $\mathbf{B}_d$ is the distortion introduced by the circuit. $\mathbf{B}_d$ appears like noise to the circuit (zero-mean and uncorrelated to the input signal) and its properties are described by the $n \times n$ covariance matrix $\mathbf{C}_d$. The fact that we can consider the distortion introduced by a circuit as noise gives rise to the BLA-based DCA:

1) Excite the circuit with the proper excitation multisine that represents the class of signals the circuit will encounter in its application.
2) Calculate the BLA of each sub-circuit and use it to calculate the distortion introduced by each sub-circuit
3) Apply a noise analysis to the distortion sources in the circuit to obtain the total distortion generated by the circuit.

When the circuit is behaving dominantly linear (like an op-amp in feedback) the BLA will lie very close to the small-signal behaviour of the circuit. This small-signal behaviour can be extracted quickly using AC simulations. In the remainder of this paper we will assume that the BLA is equal to the small-signal behaviour. A validity test, which gives an indication of the error made with this assumption, will be introduced in Section II.

[1]Waves in the analysis, because the S-parameters of degenerate circuits like an open, short circuit or a through connection exist in contrary to the voltage and current-based representations like Y and Z parameters, which become innite or zero in the degenerate cases

C. Applying a noise analysis to distortion sources

The noise analysis detailed in [1] allows to calculate the distortion covariance matrix of the total circuit $\mathbf{C}_t$, starting from the known distortion covariance matrices of the different stages. The application of this algorithm in a DCA was first proposed in [5]. The noise analysis requires the BLA of each sub-circuit, a matrix that indicates how the sub-circuits are interconnected and the covariance matrix of the internal distortion sources. The way these matrices are constructed is detailed below. The BLA-based DCA is performed at each frequency of interest. For readability, the frequency dependency (f) will be omitted. The BLA of each of the s sub-circuits are gathered in a block-diagonal matrix.

$$\mathbf{S}_{BLA} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{S}_{BLA,1} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{S}_{BLA,s} \end{bmatrix} \tag{3}$$

In a circuit with p external ports, the first p rows and columns of $\mathbf{S}_{BLA}$ are equal to zero.

The interconnection of the sub-circuits is represented in the algorithm by the interconnection matrix $\mathbf{\Gamma}$ [1]. This matrix is zero everywhere, except for a 1 at places where ports of sub-circuits are connected together.

The covariance matrix of the distortion has the following shape:

$$\mathbf{C}_d = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbb{E}\left\{\mathbf{B}_D\mathbf{B}_D^{\dagger}\right\} \end{bmatrix} \tag{4}$$

Again, the first p columns and rows of the matrix are equal to 0. $\mathbb{E}$ denotes the expected value and $\dagger$ the Hermitian transpose. $\mathbf{B}_D$ is the stacked column vector of distortion sources $\mathbf{B}_{d,i}$ for each of the s sub-circuits in the analysed circuit:

$$\mathbf{B}_D = \begin{bmatrix} \mathbf{B}_{d,1}^{\top} & \mathbf{B}_{d,2}^{\top} & \dots & \mathbf{B}_{d,s}^{\top} \end{bmatrix}^{\top} \tag{5}$$

where $\top$ denotes the transpose of a vector

In the first step of the algorithm, the inverse of the connection scattering matrix $\mathbf{W}$ is calculated and it is cut into four pieces

$$\mathbf{W}^{-1} = (\mathbf{\Gamma} - \mathbf{S}_{BLA})^{-1} = \begin{bmatrix} \mathbf{W}_{tt} & \mathbf{W}_{ts} \\ \mathbf{W}_{st} & \mathbf{W}_{ss} \end{bmatrix} \tag{6}$$

$\mathbf{W}_{tt}$ is a square matrix of size p. In the second step, the distortion covariance matrix of the total circuit $\mathbf{C}_t$ is obtained as

$$\mathbf{C}_t = \begin{bmatrix} \mathbf{W}_{tt} & \mathbf{W}_{ts} \end{bmatrix} \mathbf{C}_d \left(\begin{bmatrix} \mathbf{W}_{tt} & \mathbf{W}_{ts} \end{bmatrix} \right)^{\dagger} \tag{7}$$

the distortion contributions to the total distortion generated by the circuit can be obtained by writing the expressions for the different elements in $\mathbf{C}_t$ as products of vectors [5].

II. Evaluation of the accuracy of the method

Remember that the approximation can be used that the BLA is equal to small-signal behaviour of the circuit. A test is needed to indicate the validity of this approximation. To test the accuracy of the DCA, the distortion at the ports of the total circuit is compared to the distortion predicted by the DCA (Figure 2).

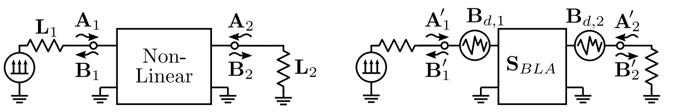


Fig. 2. To test the DCA, the distortion at the waves of the ports of the total circuit $\mathbf{A}_i$ and $\mathbf{B}_i$ is compared to the predicted distortion by the DCA $\mathbf{A}'_i$ and $\mathbf{B}'_i$

A. Distortion of the total circuit

The distortion at the $\mathbf{A}$ and $\mathbf{B}$ waves of the total circuit is determined using the BLA from the multisine source to each of the waves. If the total system is a n-port, $2n$ Single-Input Single-Output (SISO) BLAs are required. Calculating a SISO BLA (without approximation) doesn't require the special excitation signals needed to estimate the BLA of the sub-circuits and can be done quickly [8]. The SISO BLAs are used to determine the actual distortion added by the circuit in the following way:

$$\begin{aligned} \mathbf{Z} &= \begin{bmatrix} \mathbf{B}_{d,t} \\ \mathbf{A}_{d,t} \end{bmatrix} = \begin{bmatrix} \mathbf{B}_t \\ \mathbf{A}_t \end{bmatrix} - \begin{bmatrix} \mathbf{G}_{MS\to\mathbf{B}} \\ \mathbf{G}_{MS\to\mathbf{A}} \end{bmatrix} \cdot MS \\ \mathbf{C}_{BA} &= \mathbb{E}\left\{\mathbf{Z}\mathbf{Z}^\dagger\right\} \end{aligned} \tag{8}$$

$\mathbf{B}_t$ and $\mathbf{A}_t$ are the waves at the ports of the total circuit, MS is the excitation signal and $\mathbf{G}_{MS\to\mathbf{A},\mathbf{B}}$ are the column vectors of stacked SISO best linear approximations from multisine to the $\mathbf{A}$ and $\mathbf{B}$ waves. $\mathbf{C}_{BA}$ contains all information about the distortion around the total circuit.

B. Distortion predicted by the DCA

The covariance matrix of the distortion in a second way, using the results of the DCA. Both covariance matrices should be equal, any difference will be due to the approximation made in the DCA. If the load impedances seen from the total circuit are linear, we can calculate the distortion at the A and B waves as follows:

$$\begin{aligned} \mathbf{W}_t &= \left(\begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{I} & \mathbf{0} \end{bmatrix} - \begin{bmatrix} \mathbf{L} & \mathbf{0} \\ \mathbf{0} & \mathbf{S}_{BLA,t} \end{bmatrix} \right) \quad \mathbf{C} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_t \end{bmatrix} \\ \mathbf{C}'_{BA} &= \mathbf{W}_t^{-1}\mathbf{C}\left(\mathbf{W}_t^{-1}\right) \end{aligned} \tag{9}$$

where $\mathbf{S}_{BLA,t}$ is the Best Linear Approximation of the total circuit and $\mathbf{L}$ is the S-matrix that contains the reflection factors seen from the total circuit. $\mathbf{C}_t$ is the distortion covariance matrix of the total system, as calculated in (7). Comparing $\mathbf{C}_{BA}$ from (8) and $\mathbf{C}'_{BA}$ from (9) will allow to estimate the error in the DCA.

III. Extension to include differential signals

Extending the BLA-based DCA to use differential signals is straightforward because the BLA-based DCA is based on techniques from linear network theory. Adapting the DCA to handle differential signals requires a change to the distortion covariance matrix (4) and the block diagonal matrix which contains the BLA of the sub-circuits (3). The waves at differential ports are transformed into common-mode and differential-mode waves [7].

$$\begin{aligned} A_{dm} &= (A_1 - A_2)/\sqrt{2} \qquad & B_{dm} &= (B_1 - B_2)/\sqrt{2} \\ A_{cm} &= (A_1 + A_2)/\sqrt{2} \qquad & B_{cm} &= (B_1 + B_2)/\sqrt{2} \end{aligned}$$

where A_{dm} and B_{dm} are the differential mode A and B-waves at the two ports, A_{cm} and B_{cm} are common-mode waves. This transformation changes the reference impedance of the waves, but that doesn't introduce problems, as long as the BLA of each sub-circuit is transformed correctly into its mixed-mode S-parameters with the matrix transformations described in [7]. The modified DCA will now give a differential and common-mode contribution to the output, which are easier to interpret.

IV. Examples: Two-stage CMOS op-amps

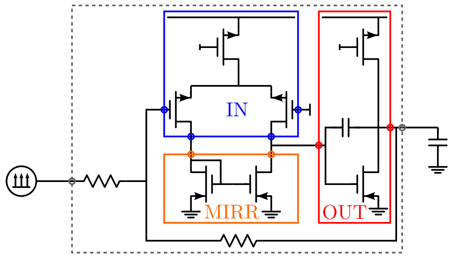
As an example, the BLA-based DCA is applied to two different two-stage operational amplifiers: a Miller op-amp and a Folded-cascode op-amp (Figure 3a). The op-amps are designed in an 180nm CMOS process to drive a load capacitance of 10 pF with a gain-bandwidth product of 10MHz. Both op-amps are placed in an inverting feedback configuration with a gain of 4 and excited by a multisine with excited frequencies up to 10MHz. The multisine excites only odd frequency bins, to allow a separation between the odd and even order non-linear distortion [8]. The simulations were performed in Keysight's Advanced Design System (ADS). 50 phase realisations of the multisine were simulated to be able to obtain a good estimate of the distortion covariance matrices. The output wave of the different simulations is shown in black on Figure 3d. The response to the multisine is very linear, with a signal to distortion ratio of about 70dB.

The distortion contributions are hard to interpret when the DCA is applied without considering the fact that the signals at certain ports are differential (Figure 3b). Cancelling contributions appear at the output of the total circuit, obscuring the actual dominant source of non-linear distortion. Applying the modified DCA, where the sub-systems are represented with their mixed-mode S-parameters yields a better image of what's going on in the circuit (Figures 3c). Distortion at the differential mode of the differential pair is the dominant source of distortion in both op-amps.

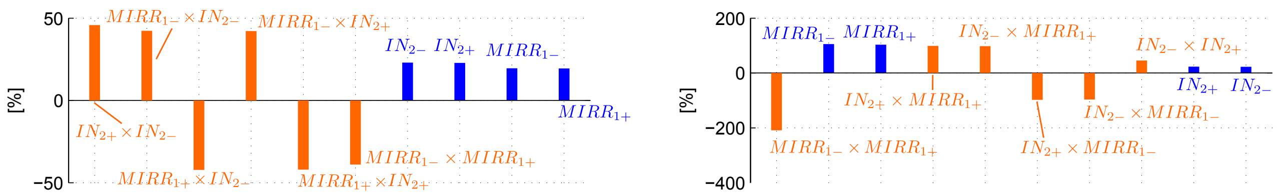
The accuracy of the DCA is high, as indicated by the small difference between $\mathbf{C}_{BA}$ and $\mathbf{C}'_{BA}$ shown in blue and red respectively in Figure 3d.

V. Conclusion

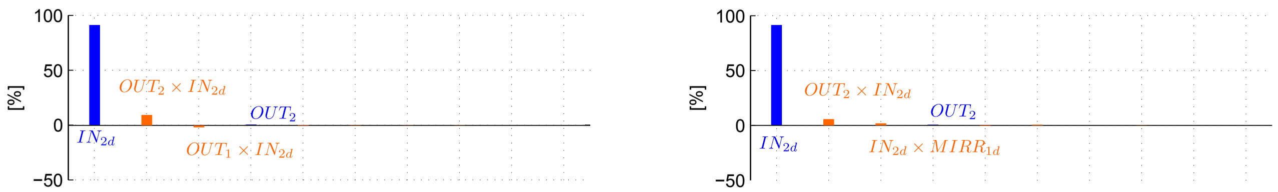
It was shown that the DCA based on the BLA can be extended to deal with differential signalling in analog circuits. Using the mixed-mode representation of the sub-circuits during the analysis results in non-linear contributions that are easier to interpret than in the classical case. Determining the BLA of all the sub-circuits requires a large amount of simulations. To avoid this large simulation time, the assumption is made that the BLA can be replaced by the small-signal behaviour. It is shown that the error made in that assumption is indicated by the difference between the distortion at the output of the circuit and the sum of contributions. The examples show only a small error in the DCA.



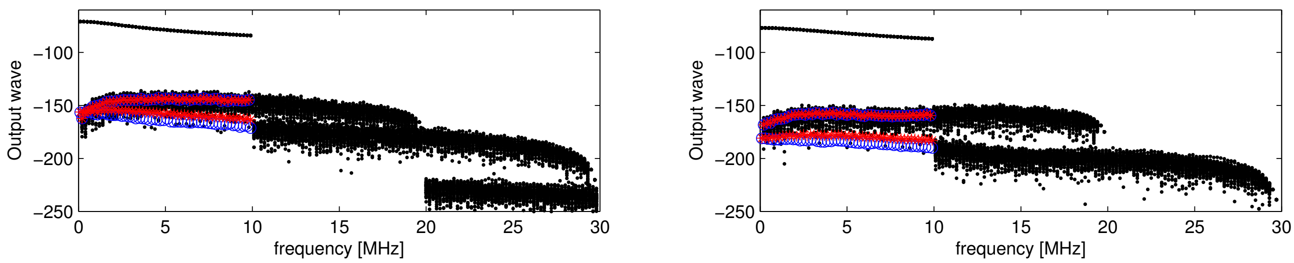
(a) The op-amps under test are divided into different blocks. The Miller op-amp is shown on the left and the Folded-Cascode on the right. The inputs consist of a differential pair labelled IN. The differential current of the IN stage is converted back to a single-ended voltage by a current mirror (MIRR). The second stage of the op-amp is labelled OUT. The total circuit consists of the op-amp in its inverting feedback configuration, as indicated by the dashed lines.



(b) Using a normal representation, the results of the DCA are hard to interpret. Large cancelling contributions appear at the output due to the differential nature of the signals internally in the circuit. These large contributions obscure the actual dominant source of distortion. The blue bars indicate contributions originating from one of the ports in the circuit, while the orange contributions are due to correlation between different distortion sources in the circuit. The DCA is performed at $5\mathrm{MHz}$



(c) Using the mixed-mode representation in the DCA yields results that are easier to interpret. we can clearly see that the input stage is by far the dominant source of distortion in both op-amps.



(d) The error made in the DCA is quite small as is indicated by the small difference between the simulated distortion (○) and the total distortion predicted by the DCA ($*$) at the output wave of the op-amps. In black, the actual response of the circuit to the multisine excitation is shown.

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