Determining the Dominant Nonlinear Contributions in a multistage Op-amp in a Feedback Configuration

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Abstract—**In this paper a simulation based method is proposed to determine the position of the dominant nonlinear contribution in the schematic of multistage op-amp operated in a feedback configuration. The key idea is to combine the Best Linear Approximation (BLA) and a classical noise analysis to determine the dominant source of nonlinear contributions. This results in a powerful yet simple design tool which does not require special analyses or custom models. As an example, the method is applied to a folded-cascode op-amp.**

Index Terms—**nonlinear distortion, operational amplifiers**

I. Introduction

MOST analog design flows rely only on linear time invariant reasoning while designing an analog/RF circuit. When the linear design flow is completed, the importance of nonlinearities is assessed by identifying compression points or intercept points. Those provide a measure of the nonlinear behavior of the total circuit only. This standard approach does not give any clue to help the designer to modify the design to decrease the nonlinearities as no information is provided about the source of the nonlinear distortion.

In [1]–[3] a Volterra-based approach was used to localize the nonlinearity of the circuit in an analytic way. For larger circuits, this analytic method yields lengthy complex expressions. Overview is hence easily lost. Those methods also require the replacement of the transistor model by an approximate analytic-nonlinear model.

In this paper, a method is proposed which can be positioned in between the linear design framework and the symbolic Volterra theory. The op-amp is considered to consist of a cascade of two or more gain stages. Each stage is considered as a black box. No knowledge about the interior of a stage is used. The nonlinear distortion is determined by a transient analysis. The input and output signal of every stage is measured during the simulation. The only constraint imposed on the transient simulation is the choice of the excitation signal used. A so called multisine excitation allows one to determine the BLA of the system. The BLA consists of a Frequency Response Function (FRF) model and a colored power spectral noise source to model the influence of the nonlinearity [5]. One can hence consider the nonlinear distortions as an additional colored noise source. Using regular noise analysis now allows one to determine the distortion that is introduced by every stage.

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Section II explains the method and the theory behind it in more detail. Then, in Section III, the method is applied to a folded-cascode op-amp.

II. Methodology

In this section we explain the theory behind the method. First we define the multisine excitation signal that is used in the transient analysis. Using a special multisine allows to split the even and odd nonlinear contributions. They can be “measured” separately. Second, the theory of the BLA is introduced. This leads to the description of the nonlinear contributions as a colored Gaussian noise source. Finally we apply the noise analysis on the cascaded stages.

A. Multisine excitation

In [4] an odd random-phase multisine is shown to be a well-suited excitation signal for the detection of nonlinearities in a measurement context. This claim remains valid for simulations. Random-phase multisines combine the best of random excitations and periodic signals.

- Random excitations are close to real world signals. They allow a broad measurement bandwidth at the cost of spectral leakage and a reduced signal-to-noise ratio at some frequencies. They also hamper an easy detection of the nonlinearity.
- Periodic signals have a deterministic spectrum. They do not mimic real world signals very well. When properly designed, they don’t suffer from spectral leakage and can ease the detection of the nonlinearity.

A random phase multisine behaves as a random noise signal that mimics real world signals but comes with the high signal-to-noise ratio and the nonlinear detection capability of a periodic signal. A random-phase multisine with N components is described by:

$$s(t) = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} A_k \cos\left(2\pi k f_0 t + \phi_k\right) \quad (1)$$

where A_k and ϕ_k are the amplitude and phase of the k^{th} spectral line and f_0 is the resolution frequency of the multisine. The value of the phase spectrum is the result of a uniform random process over $[0, 2\pi[$. By imposing additional constraints on the frequency grid, it is possible to construct a multisine suited for the analysis of nonlinearities: the odd random phase multisine [5].

In this signal, only odd frequency bins are present ($A_{2k} = 0$). An even nonlinearity produces components at a frequency

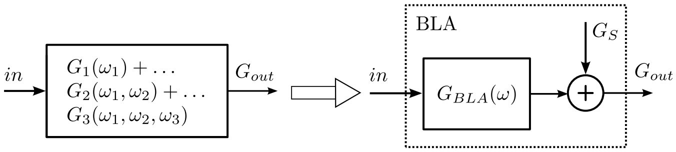

Figure 1. Applying the BLA on a nonlinear stage

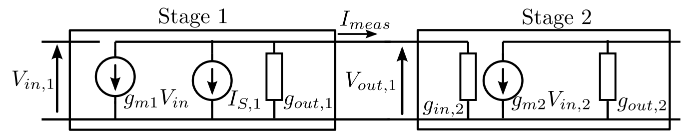

Figure 2. Cascade of two stages with a finite input impedance

which is the sum of an even number of excited frequencies. As the excited frequencies are all chosen to lie on an odd frequency grid, the sum of an even number of such frequencies will lie on an even grid. This means that even nonlinearities will not interfere with the response of the system at the excited frequencies. Hence an even distortion becomes measurable by looking at the spectrum present at the even frequency grid lines.

The odd order nonlinear contributions will always contribute to the excited frequency lines. The sum of an odd number of odd frequencies always yields an odd frequency.

Two approaches exist to determine the level of odd nonlinearities [6]. In this paper the faster of both is used. Some odd frequencies are omitted in the excitation signal. One then interpolates the measured distortion spectra at the non-excited frequency lines (also called the “detection lines”) to estimate the odd non-linear contribution present at the excited lines [6]. To choose which excitation frequencies to omit, the excited bins are grouped into groups of 4 neighboring excited bins from which one bin is randomly removed.

B. Best Linear Approximation

Linear system theory describes the response of an LTI system as

$$Y(f) = G_0(f) \cdot U(f) \tag{2}$$

with $U(f)$ and $Y(f)$ respectively the deterministic linear input and output spectra and $G_0(f)$ the Frequency Response Function (FRF) of the system. For nonlinear systems, this relation is no longer generally valid, but can be used to approximate the linearized behavior of the system around an operating point in least squares sense. This approximation is called the Best Linear Approximation (BLA). For a random multisine excitation with a sufficiently large number F of excited frequencies, the FRF can be written in the form [6]

$$G(f) = G_{BLA}(f) + G_S(f) + G_N(f) \tag{3}$$

with:

- $G_{BLA}(f)$ the best linear approximation. It consists of the linear term $G_0(f)$ and a systematic nonlinear bias term $G_B(f)$ which describes the compression/expansion of the system and is caused by odd nonlinearities,
- $G_S(f)$ the stochastic nonlinear contribution which acts as a noise source with zero mean,
- $G_N(f)$ the simulation (or measurement) noise.

This BLA represents the response of the nonlinear system to signals with similar properties (e.g. same power spectrum, probability density function, ...) as the signal applied to determine the BLA.

The BLA assumes that the system consists of an FRF $G_{BLA}(f)$ with an additive output noise source G_S which accounts for the stochastic nonlinear contributions (see Figure 1). Since the stochastic nonlinear contributions act like noise, it is possible to apply techniques borrowed from classical noise analysis on these nonlinear contributions. When we apply the BLA to every stage of the op-amp, we get a (nonlinear) noise source $G_S(f)$ for every stage. If we refer all the (nonlinear) noise sources in the system to the output, we can compare their contribution to the total measured output distortion of the system.

C. Determining and comparing the nonlinear contributions of each stage in an Op-amp

In order to refer the (nonlinear) noise contribution of each noise source to the output node, we need to know:

1) the power spectral density (PSD) of each noise source
2) the FRF between that noise source and the output node

If the stages behave dominantly linear, it is possible to perform the noise analysis using AC FRFs only. From now on, we will neglect the nonlinear bias term $G_B(f)$. To verify the validity of this assumption, it is sufficient to compare the (noisy) G_{BLA} obtained by the division of the spectra calculated during the transient analysis to the noise-free AC FRF G_0. The trade-off to be made is a classical bias versus variance trade-off. We have chosen to allow for a bias of a few dB in the amplitude and a few degrees in the phase in this paper.

1) Determining the PSD of the noise source: ideal case: To determine the PSD of the (nonlinear) noise source for a certain stage, we calculate the difference between the simulated nonlinear response of that stage and its linearized response. Looking at Figure 1 we find that in general:

$$G_S = G_{out} - G_{BLA} \cdot in \tag{4}$$

If we neglect the nonlinear bias term G_B, G_{BLA} boils down to the AC FRF G_0. If the input and output loading impedance of the stage are infinite, we can use the voltages measured at the input and the output port alone. This results in the ideal behavior

$$G_{S,i} = V_{out,i} - \mathrm{G}_{0,i} \cdot V_{in,i} \tag{5}$$

where $G_{S,i}$ is the (nonlinear) noise contribution, $V_{in,i}$ and $V_{out,i}$ are the voltages measured at the input and output of the i^{th} stage during the transient analysis respectively and $G_{0,i}$ is the FRF of the i^{th} stage, calculated with an AC analysis.

2) Determining the PSD of the noise source: real world case: In an op-amp, it’s not possible to consider the input impedance of a stage to be infinite. Expression 5 will therefore yield a poor approximation of the PSD in this case. When the input impedance of the loading stage is not infinite, theory requires us to apply a full two-port noise analysis. To avoid the complexity, we have chosen an intermediate solution. We use the Norton equivalent for the output of the stage-under-test and consider the (nonlinear) noise source to be a current source only. This neglects the voltage noise source that is present in the full 2-port case [7].

Consider a cascade of two stages as shown in Figure 2. Determining the noise contribution $I_{S,1}$ can be done using the following formula:

$$I_{S,1} = \frac{g_{in,2} + g_{out,1}}{g_{in,2}} \left(I_{meas} - \mathrm{FRF}_{V_{in,1} \rightarrow I_{meas}} V_{in,1} \right) \quad (6)$$

where $g_{in,2}$ is the input conductance of stage 2 and $g_{out,1}$ is the output conductance of stage 1. I_{meas} is the current flowing out of stage 1 and into stage 2 and $V_{in,1}$ is the input voltage of stage 1. Both are measured during the transient analysis. $\mathrm{FRF}_{V_{in,1} \rightarrow I_{meas}}$ is the transconductance of the first stage. All conductances and the transconductance are determined using an AC analysis.

3) Determining the FRF between the source and the output: The FRF needed to refer the calculated noise contribution to the output is determined using another AC analysis. An AC source is placed at the assumed location of the (nonlinear) noise source and its response to the output is calculated.

- For nonlinear contributions calculated using (5), an AC voltage source is placed in series with the considered stage.
- For nonlinear contributions calculated using (6), an AC current source is placed between the output of the considered stage and AC ground.

D. Simulations

The necessary simulations were performed using classical AC and transient analysis, while the post-processing of the data was done in MATLAB.

First, the multisine excitation signal is generated in MATLAB and then imported into the transient simulation as a time-domain waveform. The sampling frequency of the simulation is chosen to be 10 times the maximum frequency of the multisine.

For an n-stage op-amp, the following simulations are needed:

1) A transient analysis with a multisine excitation to determine the nonlinear contributions at the unexcited frequency bins of the multisine. The op-amp can be placed in an inverting feedback configuration.
2) One AC analysis to determine the FRF of the stages. A voltage to current FRF is needed for the stages which are followed by a stage with a finite input impedance. A voltage to voltage FRF is obtained for the stages with an infinite load. This analysis can also be used to determine the input impedance of the stages, by measuring the current flowing into the stage.
3) For each stage, an AC analysis is needed to determine the total conductance of it's output node. For this AC analysis, the input of the stage is AC grounded.

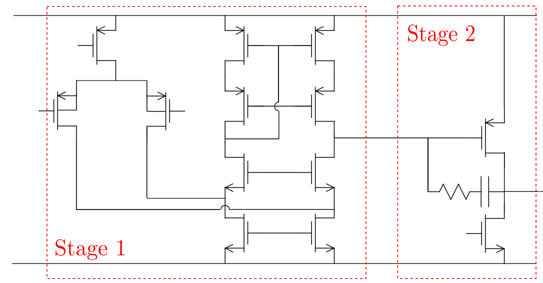

Figure 3. Op-amp used for the simulations

4) For each stage, an AC analysis to determine the FRF from the considered noise source to the output. An AC source is added at the location of the equivalent nonlinear noise sources and its influence is measured at the output.

The set of AC analyses is not only used to determine the FRF of the subsystems, but also their input and output impedance and the impact of the different nonlinear contributions to the output. The latter enables the use of this nonlinear analysis in a classical noise analysis. The relative importance of each nonlinear source is obtained assessing its relative contribution to the total nonlinear distortion at the output. This results in an easy to use analysis tool to determine the dominant sources of nonlinear distortion.

The AC analyses are performed up to the sample frequency of the analysis, with a resolution determined by the lowest frequency of the multisine.

For the transient analysis, a fixed time step is chosen in function of the sampling frequency. Two periods of the multisine are simulated. The first period is discarded to suppress transient effects. The integration method is trapezoidal to prevent artificial damping of the poles in the op-amp, such that the results of the transient analysis match the results of the AC analysis up to the frequency where warping starts to occur [8].

III. Example: Folded-Cascode Op-amp

As an example, the developed method is applied to the folded-cascode op-amp shown in figure 3. The op-amp is designed for the UMC.18 CMOS technology. During the simulations, a BSIM3v3 model is used for the MOSFETs. The op-amp under test has a gain bandwidth product of 100 MHz and a DC gain of 80 dB. It is connected as an inverting amplifier with a gain of 10. The impact of the resistive loading of the output stage is reduced by a voltage buffer inserted between the output and the feedback resistor. Since the analysis method uses the BLA to represent the stages, and since no inside information of the stages is needed, the method can easily be expanded to be used on a higher level architecture or on other op-amp architectures.

The multisine used for the experiment has a resolution of 100 Hz and excites frequencies up to 10 MHz. The sample frequency of the simulation is 100 MHz. The phase spectrum of the multisine is random. Note that a set of 100 realizations of the multisine was used to select the signal with the smallest

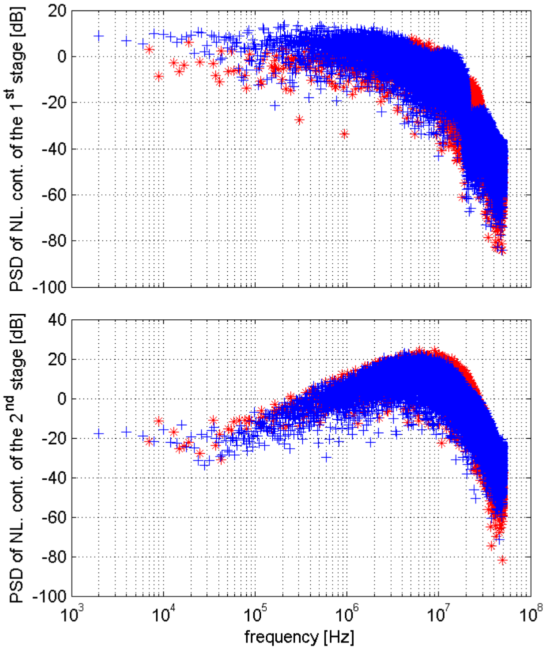

Figure 4. Output referred nonlinear contributions of the first and second stage. (+) are the contributions at the even frequency bins and (∗) are the contributions at the odd frequency bins.

crest factor. Its amplitude is scaled such that the output covers 80% of the supply voltage. This prevents clipping and imposes that the stages behave dominantly linear. The linear FRF, determined with the AC analysis can therefore be used.

The following results will be discussed: first, we analyze which stage contributes most to the nonlinear distortion. Next, it is shown that the total distortion is equivalent to the sum of all distortion contributions.

The calculated nonlinear output contributions of both stages are shown in Figure 4. Blue symbols show the contributions at the even spectral lines, representing the even nonlinear distortion. Red symbols represent the contribution at the odd spectral lines without excitation, representing the odd nonlinear contributions. At low frequencies, the first stage is the dominant source of nonlinear distortion. At frequencies close to the gain bandwidth product, the second stage is responsible for most of the distortion.

To verify whether the contributions are correct, the sum of both output referred contributions is compared to the actual measured output spectrum during the transient simulation. The result of this comparison can be seen in Figure 5. The difference between the sum of the calculated contributions and the measured distortion at the output gives a measure for the error level of the procedure. Because the sample frequency of the transient analysis is $100\,\mathrm{MHz}$, the results can only be considered to be accurate in a frequency range of up to about $10\,\mathrm{MHz}$. The main source of errors is the fact that the AC FRF is used during the analysis and not the BLA of the stages. Second, the SISO representation of a two-ports problem will introduce errors. The last source of errors in the analysis is due to numerical precision. The measured current between the stages is used to calculate the nonlinear contribution of the

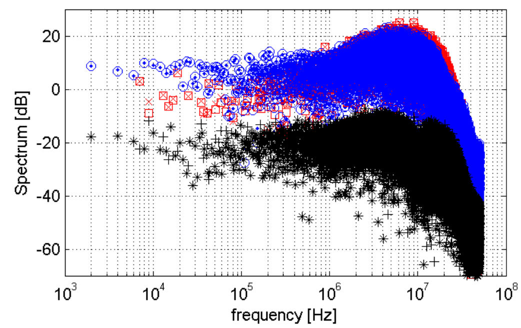

Figure 5. Comparison between the sum of the calculated output referred nonlinear contributions and the actual output spectrum measured during the transient simulation. (□) and (◯) represent the measured output distortion at odd and even frequency bins respectively. (×) and (·) represent the sum of the calculated nonlinear contributions of both stages at odd and even bins respectively. (+) and (∗) represent the difference between both at odd and even bins respectively.

first stage. At very low frequencies, the input impedance of the second stage is very large. Hence the current becomes very small and the numerical precision of the calculations comes into play and this increases the error.

IV. Conclusions

A transient simulation using a multisine excitation allows the extraction of a "best" linear transfer function and an equivalent nonlinear "noise" source. If the system behaves dominantly linear, one can use the AC analyses to determine the output distortion generated by each stage. Assuming the nonlinear distortion behaves as a current noise source allows to take finite input impedance of the stages into consideration without using a full two-port noise analysis. The calculated distortion is then referred to the output by simulating the AC transfer function between the assumed source and the output. By doing so, the nonlinear contribution of each stage in an op-amp to the output is determined. This method allows to determine the dominant source of nonlinearities without using special simulation techniques or models.

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