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Distortion Contribution Analysis by combining the Best Linear Approximation and noise analysis

Adam Cooman, *Student Member, IEEE,* and Gerd Vandersteen, *Member, IEEE.*

***Abstract*—The non-linear performance of analogue electronic circuits is crucial during the design phase, while circuit simulators only give measures about the distortion generated by the total circuit, leaving designers clueless about the source of the problem. Distortion Contribution Analysis (DCA) is a simulation-based analysis technique that determines the distortion generated in the sub-circuits and shows their contribution to the total distortion of the circuit. DCA can be used to efficiently decrease the distortion generated by a circuit, because it points the designer to the origin of the problem.**
Recently, a DCA based on the Best Linear Approximation (BLA) has been introduced as alternative to the Volterra-based techniques. However, a major drawback of the current implementation of the BLA-based DCA is its limitation to single-input single-output frequency response functions to model the behaviour of the sub-circuits. This approach ignores the input and output impedance of the stages, and hence introduces errors. In this paper, an extension of the BLA-based DCA is proposed which uses a MIMO port representation of the sub-circuits. Combining the port representation with a multi-port noise analysis allows the analysis of non-linear circuits without adapting the modelling of the sub-circuits.

I. Introduction

The non-linear performance of analogue electronic circuits is a matter of ever-increasing importance. However, classic figures of merit that describe the non-linear performance of circuits like intercept points (IP3) or intermodulation products (IM3) only give a measure for the distortion generated by the complete system, giving no indication on how to reduce the distortion generated in the circuit.
A technique is needed that allows to pinpoint the dominant source of distortion. This enables the designer of the circuit to quickly improve the performance of his circuit. Such an analysis technique will be called Distortion Contribution Analysis (DCA) in this paper.

A first approach to implement a DCA is based on Volterra theory [1]–[3]. The Volterra-based DCA works well for smaller circuits which contain only a small number of static non-linear elements. The amount of terms rises quickly for larger networks and the results become impossible to interpret.

Recently, an alternative DCA has been demonstrated: instead using the full non-linear description of the circuit, linear approximations of the sub-circuits (the so-called Best Linear Approximation (BLA) models [7]) are used and combined with classical linear noise analysis techniques. The use of a wideband excitation signal makes it possible to consider the non-linear distortion introduced by a sub-circuit as noise. Combining the BLA with a noise analysis enables to determine the distortion contributions of all the sub-circuits to the distortion generated by the complete system.
The BLA-based DCA has already been demonstrated on two-stage operational amplifiers [4], on a discrete-time sigma-delta analog-to-digital converter [5] and on a complete receiver [6]. However, a major drawback of the previous implementations of the BLA-based distortion contribution analysis is in its limitation to single-input single-output (SISO) linear approximations to model the behaviour of the sub-circuits. The SISO approach does not allow to include the input and output impedance of stages which creates errors at higher frequencies.

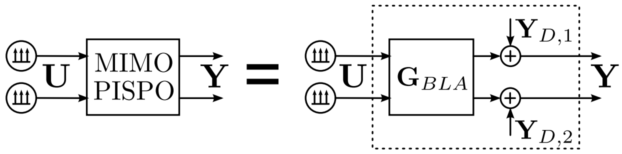

Fig. 1. The response of a MIMO non-linear system to a multisine can be approximated in least squares sense by a linear system ($\mathbf{G}_{BLA}$) where the distortion generated by the circuit is modelled as noise ($\mathbf{Y}_D$)

In this paper, we propose an extension of the BLA-based DCA which uses port representations of the sub-circuits. The use of port representations leads to the use of a multiple-input multiple-output (MIMO) BLA which is combined with an existing multi-port noise analysis [8].

This paper starts by summarising the basics of the MIMO BLA (Section II) and the multi-port noise analysis (Section III). Then, the novel MIMO BLA-based DCA is described in Section IV. Finally, the method is applied to a shunt feedback low-noise amplifier (section VI).

II. Basics on the Best Linear Approximation

The class of systems will be limited to the time-invariant period-in same period-out (PISPO) systems. The PISPO class includes a wide class of dynamic non-linear systems, but excludes non-linear systems that generate sub-harmonics or chaotic systems. The inputs of the system are excited with wideband signals that have a fixed power spectral density (PSD) and probability density function (PDF). The phase of the input signals is random.

The relation between the input vector $\mathbf{U}$ and output vector $\mathbf{Y}$ of a PISPO system excited with the wideband excitation signal can be approximated in least squares sense at each frequency f by the Best Linear Approximation [7] where

Adam Cooman and Gerd Vandersteen are with the department ELEC at the Vrije Universiteit Brussel (VUB), Brussels, Belgium.
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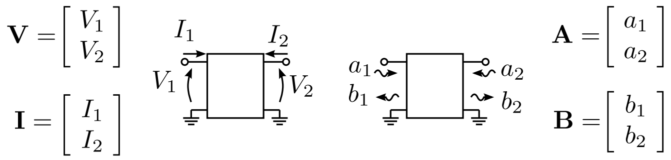

Fig. 2. Definition of the voltages, currents, waves and their corresponding vectors applied to a two-port system.

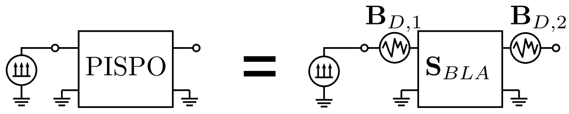

Fig. 3. Applying the BLA to a two-port circuit described with S-parameters gives a best linear S-matrix and two outward radiating noise sources.

the distortion introduced by the system is represented by output noise sources (Fig. 1). Ignoring the measurement noise, because the analysis is simulation-based, we obtain

$$\mathbf{Y}(f) = \mathbf{G}_{BLA}(f) \cdot \mathbf{U}(f) + \mathbf{Y}_D(f) \qquad (1)$$

where $\mathbf{G}_{BLA}(f)$ is frequency response matrix representing the Best Linear Approximation. $\mathbf{G}_{BLA}$ depends on the PSD, on the PDF of the inputs and on the odd-order non-linear behaviour of the system under test. When the input power goes to zero, the BLA converges to the small signal frequency response matrix. Distortion, or the deviation from the best linear behaviour, is described by the vector $\mathbf{Y}_D(f)$. Determining the distortion introduced by a sub-circuit boils down to correcting the output signal with the best linear response to the input signal:

$$\mathbf{Y}_D(f) = \mathbf{Y}(f) - \mathbf{G}_{BLA}(f) \cdot \mathbf{U}(f)$$

Similar to the BLA, the distortion vector also depends on the PDF and PSD of the input signal. However, $\mathbf{Y}_D$ depends on both the even and odd non-linearities present in the circuit. The entries of the distortion vector are mutually correlated, but are uncorrelated to the input signal $\mathbf{U}(f)$ by definition of the BLA [7]. The distortion components $\mathbf{Y}_D$ can therefore be considered as independent noise contributions with respect to the random phase of the input signal. Using several phase realisations of the multisine allows to determine the covariance matrix of the non-linear distortion sources in the circuit using

$$\mathbf{C}_D(f) = \mathrm{E}\left[\mathbf{Y}_D(f) \cdot \mathbf{Y}_D^{\mathrm{H}}(f)\right]$$

where $\bullet^{\mathrm{H}}$ is the hermitian transpose and $\mathrm{E}[\bullet]$ is the expected value.

In order to keep notation simple, we will omit the frequency dependency of all the the vectors and matrices from now on.

A. Applying the BLA to a circuit

Determining the MIMO BLA of a system requires at least two simulations for each two-port sub-circuit [7]. This process can be very time-consuming. However, the small-signal frequency response function can be used to approximate the BLA without great loss of accuracy when the system is dominantly linear [4]. Small-signal port parameters can be calculated quickly in modern circuit simulators.

In this paper, the S-parameter representation of the sub-circuits will be used to model their behaviour. A wave-based approach is used because the S-parameters of degenerate circuits like an open, short circuit or a through connection exist in contrary to the voltage and current-based representations like Y and Z parameters, which become infinite or zero in the degenerate cases.

A random-phase multisine is used as wideband excitation signal because it allows strict control of its PDF and PSD, while offering several other advantages in the context of detecting non-linear behaviour [7]. A multisine signal consists of N harmonically related sine waves:

$$U(t) = \sum_{k=1}^{N} A_k \cdot \sin\left(2\pi k f_0 + \phi_k\right)$$

where f_0 is the frequency resolution of the multisine. A_k is the amplitude and ϕ_k the the phase of the of the k^{th} component of the multisine. A_k can be used to set the PSD of the multisine. When ϕ_k is drawn from a uniform distribution on the range $[0, 2\pi[$, the PDF of the multisine becomes Gaussian.

The multisine is applied to the system and the steady-state response of the circuit to the multisine is calculated with a non-linear simulation. This can be done using a transient analysis, a periodic steady state analysis, or a Harmonic Balance analysis. The voltages and currents at the ports of every sub-circuit are measured as shown in Fig. 2, transformed to the frequency domain using a discrete Fourier transform and transformed into voltage waves using

$$\mathbf{A} = \frac{(\mathbf{V} + Z_0\mathbf{I})}{2\sqrt{Z_0}} \qquad \mathbf{B} = \frac{(\mathbf{V} - Z_0\mathbf{I})}{2\sqrt{Z_0}} \qquad (2)$$

at every frequency of the multisine. The S-parameters consider the incident waves $\mathbf{A}$ as input signals and the reflected waves $\mathbf{B}$ as outputs. Applying (1) to this system gives us the following relation between incident and reflected waves (Fig. 3)

$$\mathbf{B} = \mathbf{S}_{BLA}\mathbf{A} + \mathbf{B}_D$$

where $\mathbf{S}_{BLA}$ is the best linear approximation of the circuit. The distortion sources in $\mathbf{B}_D$ are mutually correlated wave sources that radiate outwards.
The distortion sources can be obtained by subtracting the best linear response to the input from the measured output. The small-signal S-matrix $\mathbf{S}$ can be used in this correction instead of the BLA, because we assumed the system is dominantly linear.

$$\mathbf{B}_D = \mathbf{B} - \mathbf{S} \cdot \mathbf{A}$$

III. Port-based noise analysis

The proposed BLA-based DCA is based on the noise analysis described in [8]. This work considers the complete circuit as an interconnection of several multi-port circuits and assumes the S-parameters and noise covariance matrix of every sub-circuit to be known.
The noise sources in the circuit are considered as signal sources and their contribution to the total waves flowing into the termination impedances are calculated (Fig. 4). To

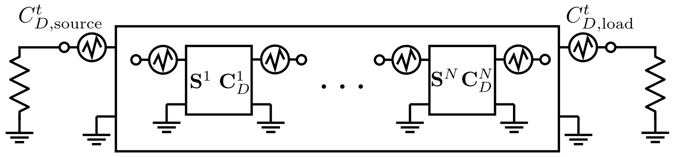

Fig. 4. In the noise analysis, the total circuit is considered as an interconnection of different noisy sub-circuits. The S-parameters and noise covariance of the sub-circuits is assumed to be known. The contributions to the noise sources of the total system $C^t_{D,\text{load}}$ and $C^t_{D,\text{load}}$ are calculated.

this end, the source and load impedances are added to the matrices describing the system. The terminal impedances are are assumed noiseless. The analysis is performed at every frequency with the following relation:

$$\mathrm{E}\left[\mathbf{A}^s\mathbf{A}^{s\mathrm{H}}\right] = (\mathbf{W}^{-1})\mathbf{C}^s_D(\mathbf{W}^{-1})^{\mathrm{H}} \tag{3}$$

$$\mathbf{C}^s_D = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}^1_D & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{C}^N_D \end{bmatrix} \qquad \mathbf{W} = \mathbf{\Gamma} - \mathbf{S}$$

$\mathbf{C}^s_D$ represents the block diagonal covariance matrix which contains the covariance matrix $\mathbf{C}^i_D$ of the N sub-circuits on its diagonal. The first few terms on the diagonal are reserved for the terminal impedances, which are assumed noiseless, so their entry is zero. $\mathbf{W}$ is the connection scattering matrix [9]. $\mathbf{S}$ is the block diagonal matrix which contains the terminal impedances (which correspond to a zero matrix) and the S-matrices $\mathbf{S}^i$ of the N sub-circuits.

$$\mathbf{S} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{S}^1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{S}^N \end{bmatrix}$$

$\mathbf{\Gamma}$ is the connection matrix which represents the interconnections between the stages.

The noise covariance matrix of the total system is determined by only looking at the incoming waves of the terminal impedances of the total circuit. Since only the output waves at the terminations are needed, we can avoid the full matrix multiplication of (3) and use only the row of the inverse of the connection scattering matrix that corresponds to the termination impedance. For example, the power of the output noise source can be written as

$$C^t_{D,load} = \mathbf{W}^{-1}_{\text{load}}\mathbf{C}^s_D(\mathbf{W}^{-1}_{\text{load}})^{\mathrm{H}} \tag{4}$$

where $\mathbf{C}^t_{D,\text{load}}$ is the power of the distortion source at the load and $\mathbf{W}^{-1}_{\text{load}}$ is the row of $\mathbf{W}^{-1}$ which contains the load. Because the expression to obtain the total noise power from the sub-circuits is now known, we can split that total power into its contributions. We re-write (4) as a product of a column vector and a row vector [11]:

$$\mathbf{C}^t_{D,\text{load}} = \left(\mathbf{W}^{-1*}_{\text{load}} \otimes \mathbf{W}^{-1}_{\text{load}}\right)\mathrm{vec}(\mathbf{C}_D)$$

where $\mathrm{vec}(\bullet)$ is the vectorisation operator which converts a matrix in a column vector by stacking its columns on top of each other. $\otimes$ indicates the Kronecker product and $*$ is the complex conjugate. The advantage of this representation is that it consists of a matrix product between a row vector and a column vector. Using point-wise multiplication instead of a matrix multiplication gives the vector with the different contributions.

IV. Distortion Contribution Analysis

In a noise analysis, it is assumed that the noise generated in different multi-ports is uncorrelated. Therefore, the noise covariance matrix $\mathbf{C}^s_D$ is block diagonal and contains the noise covariance matrix of all its components. By definition of the BLA, the distortion sources are uncorrelated to the input signal, but the distortion generated in different sub-circuits can be correlated. This leads to the use of a full covariance matrix in the noise analysis. The full matrix can be obtained by stacking the $\mathbf{B}_D$ vectors of the sub-circuits into a big column vector (5) and calculating the covariance of this vector over the different multisine realisations.

$$\mathbf{B}^s_D = \begin{bmatrix} \mathbf{B}^1_D \\ \vdots \\ \mathbf{B}^N_D \end{bmatrix} \qquad \mathbf{C}^s_D = \mathrm{E}\left[\mathbf{B}^s_D\mathbf{B}^{s\mathrm{H}}_D\right] \tag{5}$$

$\mathbf{C}^s_D$ can be used in the noise analysis as explained before. Note that B^s_D is zero mean by definition of the BLA.

V. Algorithm for the BLA-based DCA

Combining everything together leads to the following procedure to determine the different contributions to the noise covariance matrix of the total system.

1. Obtain the response of the circuit to the multisine
Apply several random phase realisations of the multisine to the complete system and determine the steady-state response of the system to each multisine. Save all the voltages and currents flowing into the ports of the sub-circuits of interest. If necessary, transform the signals to the frequency domain. Convert the signals into waves using (2)
2. Determine the S matrix of every sub-circuit
Determine the S-parameters of every block in the circuit at the frequency points determined by the multisine. Put the different blocks in the block diagonal matrix $\mathbf{S}$. Build the interconnection matrix $\mathbf{\Gamma}$.
3. Determine the distortion introduced by every sub-circuit
Determine the distortion sources of every sub-circuit k for the i^{th} realisation by removing the linear response to the input from the measured output (as was explained in section II)

$$\mathbf{B}^{k[i]}_D = \mathbf{B}^{k[i]} - \mathbf{S}^k\mathbf{A}^{k[i]}$$

Stack the obtained vectors in the big vector $\mathbf{B}^{s[i]}_D$ and calculate the covariance matrix of the distortion contributions by averaging over the simulation results of different realisations of the multisine.

$$\mathbf{C}^s_D = \frac{1}{M}\sum_{i=1}^{M}\mathbf{B}^{s[i]}_D(\mathbf{B}^{s[i]}_D)^{\mathrm{H}}$$

4. Apply the noise analysis
Finally, apply the noise analysis explained in section III.

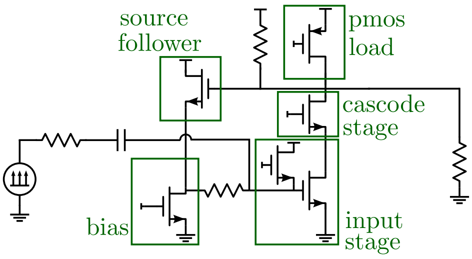

Fig. 5. Shunt feedback LNA analysed in this paper. The different circuits are marked with green boxes. A 50 Ω multisine source is used to provide the wideband excitation to the circuit.

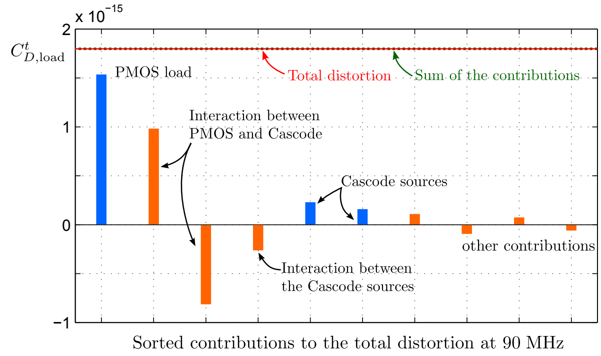

Fig. 6. Results of the DCA show that the PMOS is the dominant source of distortion. Its influence with the cascode cannot be ignored however. The total power of the output distortion source (shown as the red line at the top) is split in its contributions, which are sorted according to amplitude. Blue bars indicate the contributions originating from diagonal elements of the covariance matrix, orange bars indicate contributions from off-diagonal elements. The sum of the components (shown in green) coincides with the total distortion power.

VI. Example: Shunt-Feedback LNA

The described DCA is applied to a wide-band CMOS Low Noise Amplifier (LNA) [10] shown in Fig. 5. The wideband shunt feedback LNA is designed in Agilent's Advanced Design System (ADS) using an 180 nm CMOS process. The different sub-circuits investigated during the analysis are shown in the figure, as well as the way the wideband signal source is connected to the circuit. The multisine applied to the LNA excites frequencies between 10 MHz and 1 GHz in steps of 20 MHz. This multisine excites only odd frequency bins, which allows separation of the even and odd-order non-linear contributions [7]. Harmonic Balance simulations were used to calculate the steady-state response of the system to the different phase realisations of the multisine at the input. The small-signal behaviour of the different sub-circuits was obtained with an S-parameter simulation. The data was processed in MATLAB.

The method returns a decomposition of the non-linear distortion generated by the circuit at all the frequency bins. As an example, the result of the analysis at the 90 MHz tone of the multisine is shown in figure 6. Contributions to the total distortion are sorted by their amplitude. The complex conjugate pairs originating from the covariances in the contribution vector are summed together to obtain their actual influence on the total distortion power at the output.

The contribution originating from the PMOS load is the dominant contribution. It contributes for about 80% to the total distortion generated by the circuit at 90 MHz. The contributions on second and third place indicate a strong influence originating from the covariance between the PMOS load and the cascode stage. The other contributions in the circuit are negligible. To check whether the obtained results are reliable, the sum of the contributions is compared to the total distortion generated by the circuit. Both the total distortion and the sum of contributions are shown with lines in Fig. 6. The results of the comparison match very well.

VII. Conclusion

The MIMO BLA-based DCA described in this paper allows to find the dominant source of distortion in dominantly linear analogue circuits. It has several benefits over the previous BLA-based DCA implementations:

- It can be applied to general circuit architectures without needing to adapt the representation of the sub-circuits to the application.
- It allows to take the input and output impedance of the sub-circuits into account, significantly reducing the errors in the DCA.
- It allows to look into the correlation between the different distortion sources in the circuit.

When paired with a fast way to determine the MIMO BLA of the sub-circuits, the extended DCA could become an easy-to apply tool crucial in the design of general analogue electronic circuits.

References

[1] J. P. Aikio and T. Rahkonen. "Detailed distortion analysis technique based on large-signal voltage and current spectra" *IEEE Tran. on microwave theory and techniques*, vol. 53, no. 10, oct. 2005

[2] P. Wambacq and W. Sansen. *Distortion Analysis of Analog Integrated Circuits.* Kluwer Academic Publisher, 1998

[3] S. Cannizzaro, G. Palumbo and S. Pennisi. "Distortion Analyisis of Miller-Compensated Three-Stage Amplifiers" *IEEE Tran. on circuits and systems I: Regular papers*, Vol. 53, no. 5, may 2006.

[4] A. Cooman, G. Vandersteen and Y. Rolain. "Finding the dominant source of distortion in two-stage op-amps" *Analog Integrated Circuits and Signal Processing*, DOI 10.1007/s10470-013-0089-8

[5] G. Vandersteen and L. De Locht. "Qualification and quantification of the nonlinear distortions in discrete-time sigma-delta ADCs" *Proc. of the 2009 European conference on Circuit Theory and Design*, pp. 759-762, 2009

[6] S. Bronckers, G. Vandersteen, J. Borremans, K. Vandermot, G. Van der Plas and Y. Rolain. "Advanced nonlinearity analysis of a 6 GHz wideband receiver" *Proc. of the 2008 Instrumentation and Measurement Technology Conference (IMTC)*, pp. 1340-1343, 2008

[7] R. Pintelon and J. Schoukens. *System Identification, a frequency domain approach*, second edition, Wiley 2012

[8] J. A. Dobrowolski, "A CAD-Oriented Method for Noise Figure Computation of Two-Ports with Any Internal Topology" *IEEE Tran. on microwave theory and techniques*, vol. 37, no. 1, jan. 1989

[9] V. A. Monaco and P. Tiberio. "Computer-Aided Analysis of Microwave Circuits" *IEEE Tran. on microwave theory and techniques*, vol. 22, no. 3, mar. 1974

[10] J. Borremans, P. Wambacq, C. Soens, Y. Rolain and M. Kuijk. "Low-Area Active-Feedback Low-Noise Amplifier Design in Scaled Digital CMOS" *IEEE Tran. of solid-state circuits*, vol. 43, no. 11, nov. 2008

[11] J. Brewer. "Kronecker products and matrix calculus in system theory" *IEEE Tran. on Circuits and Systems*, vol. 25, no. 9, 1978